Fluctuation and Noise Letters
Vol. 4, No. 1 (2004) L179–L182

BLUE NOISE EFFECTS IN A NON-DYNAMICAL NEURAL MODEL SYSTEM*

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Received 23 January 2003
Revised 28 February 2003
Accepted 3 March 2003

Finite pulse width effects in level crossing detectors and similar systems, such as neurons, can yield an output noise that is, in the low frequency limit, a monotonically increasing function of the frequency. The phenomenon can well be observed at excitations with strong noise when the firing frequency is high. Blue noise effects have been observed in stochastically driven dynamical systems like harmonic oscillators. However it is surprising that it exists in a non-dynamical system like the level crossing detector.

Keywords: Neural response; colored noise; neural spike train.

1. Introduction

Threshold crossing problems of Gaussian noise are at the core of many stochastic phenomena. They play also a determining role in non-dynamical stochastic resonators in which systems first Frank Moss studied them experimentally [1]. (Note, Moss's original study is unpublished and Ref. 1 is a subsequent experimental work with his coworkers). In this paper, we show colored noise effects, blue noise, in a level crossing detector (LCD) system which was proposed by Moss to model simple neural response.

When a noise spectrum is constant versus frequency, the noise is called *white noise*. Following this fashion, a noise with $1/f^2$ spectral shape is called *red noise*, due to the strong weight lower frequencies and the 1/f noise is called *pink noise*. Thus, a noise having an increasing spectrum versus frequency is *bluish* or it can simply be called *blue noise*. In this paper, we show that a level-crossing detector with wideband input noise and

*An expanded version of the talk presented at the First SPIE Symposium "Fluctuation and Noise", entitled "Blue shot noise", at conference Noise and information in Nanoelectronics, Sensors, and Standards, Santa Fe, June, 2003; Proceedings of SPIE Vol. 5115 (2003).

wide output pulse width generates a blue noise what we call here *blue shot noise* because the similarity of this response to shot noise. The situation very similar to neural response and it follows that, under certain conditions, neural response can also produce *blue noise*. Although such blue noise effect can be seen in stochastically driven harmonic oscillators, such a system is dynamical and is governed by differential equations which can simulate differentiation and hence observing blue noise effect in dynamical systems is not surprising. However its existence in non-dynamical systems like a LCD is not a trivial problem. By observing this effect we suspect that the LCD has 'time derivative" capa bility under certain conditions.

2. Simulations

Computer simulations were carried out simulating an asymmetric level crossing detector (LCD) with the following conditions. Whenever the input amplitude at the LCD crosses a fixed threshold level from below, an output pulse of width *w* is generated. The fixed with w of output pulse correspond to a fixed time-integral of the pulse what corresponds to the case of a shot noise pulse. The correlation time τ of the band-limited white noise driving the input was one computer step. In Fig. 1, the blue shot noise effect can be seen. Apparently, for $w >> \tau$, and strong noise (rms amplitude equal or greater than the threshold) the spectrum is increasing up to a corner frequency of $f_c \approx 1/w$.

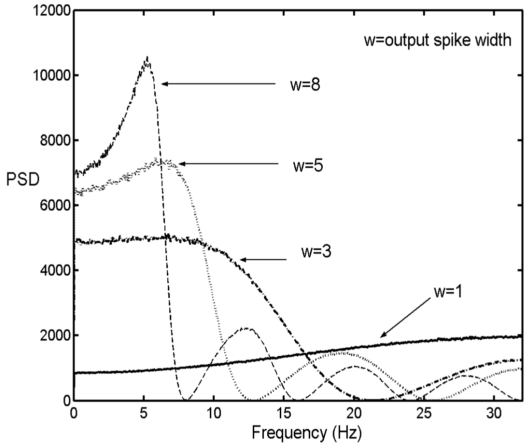

Fig. 1. The output noise spectrum of the LCD, when driven by only noise, for different values of the width of the output spike, in units of sampling time T_s=1/F_s where the sampling frequency F_s = 65 Hz. The threshold of the LCD was 1 V. The input was driven by a white Gaussian noise of variance 1 V and the input signal was absent.

On Fig. 2, at fixed pulse width *w* = 5, the dependence of the blue noise effect on the strength of the input noise is shown. Apparently, the stronger the noise the more emphasized the blue noise effect is.

On Fig. 3, at fixed pulse width *w* = 5, the dependence of the blue noise effect on the strength of additive sinusoidal input signal is shown. Apparently, the stronger the signal the more emphasized the blue noise effect is.

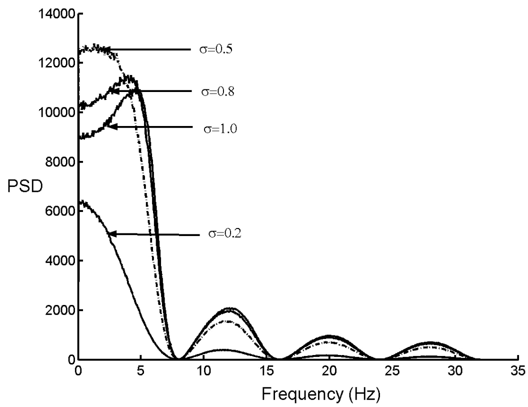

Fig. 2. Background noise spectrum at different values of the input noise level. Sampling Frequency = 65 Hz. The width of the output spike was kept constant at $5T_s$.

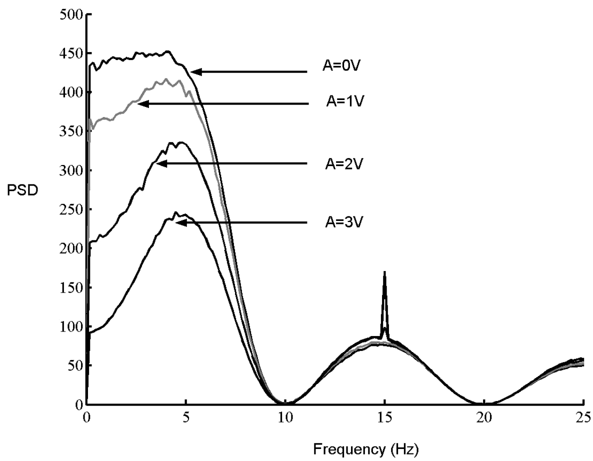

Fig. 3. The output noise spectrum at LCD threshold 1 V for different values of the signal strength at a fixed width of output spike = 5 T_s. Sampling Frequency F_s = 50 Hz. The input signal is a sinusoidal signal of frequency 5 Hz and amplitude A.

3. Explanation

When the level crossing frequency f_L (which can be evaluated from the Rice formula [2]) of the threshold level by the noise is much greater than $1/w$, the output time function is a roughly periodic spike train with mean repetition frequency $\frac{1}{w+1/f_L}$. The spike duration is fluctuation and its mean value is $1/f_L = <q>$ and in the limit $f_L \to \infty$, the spike train would be periodic with period time w , so the first harmonics would be at frequency $1/w$. As at finite f_L the period time and the pulse width are slightly fluctuating, in a random fashion, the harmonic spikes will not be sharp and they will have sidebands. The lowest sideband is the blue shot noise.

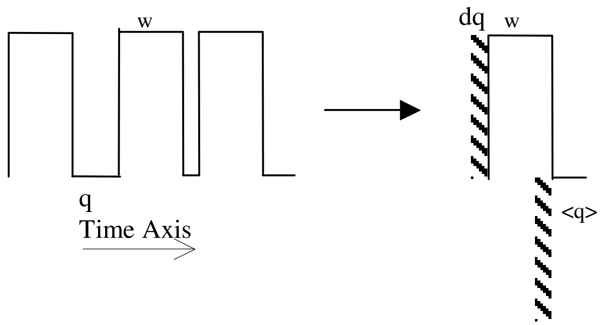

Fig. 4. Illustration of the the time-derivative characteristics of the saturated system. The shattered area, which is similar to the time-derivative of a single square pulse, is the difference between the original output pulse and the delayed one.

As we mentioned above, the blue noise effect suggests a time-derivative characteristics of the system. At the first look, it is not obvious how a time-derivative characteristics could arise in such a non-dynamical rigid system as an LCD. As seen in the above diagram, each variable-width pulse (w + q range) can be represented as a fixed-width pulse (w + <q> pulse) added to a derivative pulse shown as shattered. In this way, even a non dynamical system can simulate blue noise effect.

Acknowledgements

The research has been sponsored by the Office of Naval Research, the Army Research Lab and the TAMU Telecommunication Task Force.

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