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A stochastic resonator is able to greatly improve signal-to-noise ratio

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After a decade of doubts, first time in the history of stochastic resonance (SR), we demonstrate that a simple stochastic resonator does greatly improve the signal-to-noise-ratio (SNR) of a periodic signal with additive Gaussian noise. The particular stochastic resonator is a level-crossing detector (LCD) driven by the sum of a periodic spike train signal and a band-limited Gaussian white noise. To reach the improvement of the SNR, the stochastic resonator has to work in the strongly nonlinear response limit and the noise has to have a high cut-off frequency compared to the reciprocal duration of the spikes. We demonstrate by analog and computer simulations that the SNR gain goes beyond 4 orders of magnitude at practical conditions.

These findings get a particular importance due the fact that simplest neurone models behave very similarly to our arrangement, so the results might have direct applications in neural systems.

Stochastic resonance (SR) phenomenon [1-15] has been becoming one of the most interesting problems in the research field of *noise in nonlinear systems* due to the paradoxical fact that there is a particular amount of ”spoiling” of a signal by an additive Gaussian noise which optimises the SNR at the output of a stochastic resonator. In the case of a sinusoidal signal, the SNR is defined as the mean-square signal amplitude divided by the power density spectrum of the background noise in the vicinity of the signal frequency.

From its discovery [1] until quite recently [2], researchers had believed that SR phenomena required ”dynamical” (often bistable) systems [3-6], with proper dynamical effects in the response against an external excitation, and the internal dynamics, together with the particular nonlinearity, caused the peculiar response against noisy excitation. Recently, this view has radically been changed [2,7,8] by using (first Frank Moss) the level crossing detector (LCD) which is a stochastic resonator without internal dynamics. The LCD is a very simple unit of electronics widely used in measurement technics and FM radios. Its behaviour reminds of the firing of neurones due to the following way of response: a short uniform spike is initiated at its output whenever the voltage at its input goes through a given voltage level, the threshold voltage, in the increasing direction. During its firing the LCD is not reacting on subsequent input excitations. Feeding an LCD by the sum of a periodic signal and a Gaussian noise of variable strength yields a sharp maximum of the output SNR at a particular strength of the input noise. One important observation is that the location of the maximum of the SNR and generally the whole SNR curve is independent of the signal frequency in a wide range of frequency which proves that the level crossing dynamics of that noisy signal *inherently* contains the SR effect and the stochastic resonator (here the LCD) is not causing but only *detecting* the SR phenomenon. Another important finding about the LCD that, up to now [8], it is not only the simplest stochastic resonator but also the most effective one because its SNR is the highest among all other stochastic resonators, especially, when practical applications (including signals with a wide frequency bandwidth) are concerned [8].

An important reason why the invention of the LCD as a stochastic resonator has attracted the attention of scientists is mainly the above mentioned fact that its response behaviour is practically the same as the spike response behaviour in the simplest neurone models [9-12]. The facts that the LCD is the most effective stochastic resonator and it gives a frequency-independent response, which is a crucial condition for a good information transfer, is strongly supporting the previous conjectures that neural systems of biology might apply the SR effect to achieve an optimised information transfer.

The most important problem of the applicability of the SR effect for the improvement of information transfer is the potential possibility to increase the SNR by a stochastic resonator. During the last decade, it has been a long-standing dream of scientists that SR might be able to improve the signal-to-noise-ratio of a noisy signal. Unfortunately, all the reliable experiments and correct theories had seemingly shown that the SNR could not be improved. Sometimes, some SNR gain of a few times 10 percents had been reported at conferences, however, these results have very shortly been classified as artefacts (due to aliasing effects or other experimental errors). The most discouraging results have been published in 1995 [14] where a simple, general and exact theory proved that, in the case of linear response against the signal, SNR gain can never be produced by a stochastic resonator. It is interesting to note that the adiabatic theory [8] of an LCD predicts $SNR_{OUT}/SNR_{INP} \leq 0.85$ for the case of linear response which supports the discouraging result.

In 1995, Kiss [8] pointed out that the discouraging results are invalid for the case of nonlinear response. He has proven that an LCD driven by a random-spike-train signal (realising a case which is very similar to neural systems) and an additive Gaussian noise can improve the SNR by several orders of magnitude. He has introduced a new way [8] of determination of the output signal and noise to avoid evaluation errors due to the wideband and random input signal at nonlinear response which results in a nonlinear mixing of the frequency components of the input signal. The new determination of SNR is based on the crosscorrelation spectrum of the input signal and the total output voltage [8]. The output signal is defined as the component of the output voltage which is totally correlated with the input signal. The uncorrelated component was considered as output noise. It has been shown that the new definitions restore the classical results with periodic signal and provide less SNR at the output with random signals [8], moreover, at periodic signal the new definitions reproduce the classical results. In favour of this new evaluation method [8], it is important to note that Collins and coworkers have independently realised [10] that the case of wideband and random input signal at nonlinear response indeed requires the application of crosscorrelation functions to evaluate the SR effect in a correct way.

In the present paper, we shall avoid the use of the above mentioned new definitions by simply using a periodic spike train signal instead of randomly initiated spikes. It is well known that in the case of periodic input signal, the classical way of determination of the SNR remains valid even for the case of strongly nonlinear response limit. Therefore, we have used the same recipe to improve the SNR as has been used in [8] except, we have initiated the signal spikes periodically and evaluated the SNR in the well accepted classical way.

Now, let us describe the particular arrangement which makes a huge improvement of the SNR. The duration of the periodically repeated input signal spikes is τ_0 . The applied stochastic resonator is an LCD with a threshold level U_t , and an important condition is that the spike firing duration is chosen to be also τ_0 . The amplitude of the input signal spikes is A and the white Gaussian input noise has an rms amplitude σ and a high-frequency cutoff f_c . The following additional conditions provide the huge improvement of the SNR by our stochastic resonator [8]:

$$A<U_t \tag{1}$$

$$\sigma<<U_t\quad , \tag{2}$$

$$U_t - \sigma < A \tag{3}$$

$$1/\tau_0 << f_c\quad . \tag{4}$$

Relation (1) is to provide stochastic resonance phenomena. From Relations (1)-(3), the case of strongly nonlinear response, $\sigma<<A$, obviously follows. The role of Relation (2) is to provide a very small probability of LCD firing without input signal spike. Relations (3) and (4) provide that after the beginning of an input signal spike, the input signal and input noise together will be able to cross the threshold level U_t in the increasing direction in a very short time (typically, roughly in $1/f_c$), so an output spike will follow the input spike in a very short time. During the firing of the LCD, the LCD does not react on subsequent threshold crossings, this is why it is beneficial to choose equal duration for the signal spikes and the

LCD spikes. In this way, we can more or less reproduce the input signal at the output of the LCD and the Gaussian input noise can be removed.

Before showing the results of the detailed analysis obtained on the above described arrangement, we want to stress that the significant improvement of the SNR is very obvious even by comparing the amplitudes at the input and output with the *naked eye*. In Figure 1, a convincing demonstration of the SNR improvement by our LCD stochastic resonator can be seen. The upper curve shows the noisy signal at the input which is a mixture of a periodic spike train with an amplitude of 9 and a high-frequency Gaussian noise with an rms amplitude of 2. The noise is strong enough to make hard to recognise the original shape of the signal, though its spiky nature can be conjectured. The lower curve shows the output response of the applied LCD stochastic resonator, which has a threshold level of 10. With the present resolution of the picture, the lower record looks just identical with the original signal spike train, so it looks like *all the noise* has been removed by the stochastic resonator. The accurate analysis of long data sets shows that, though the original Gaussian input noise has been removed and the signal has been almost restored, two types of weak noises still exist. On the one hand, there is a small and random delay of the output spikes compared to the input spikes which delay represents a noise which has been called ”jitter noise” [8]. On the other hand, due to the Gaussian amplitude distribution of the input noise, sometimes accidental firing of the LCD can be observed, even when there is no input signal spike present, which is a classical type of noise in these stochastic resonators [2].

The detailed analysis of the SNR has been carried out in the classical way, where analog simulations (with real electronical LCD and physical white noise voltage) and computer simulations have given the same results. On Figure 2, the SNR gain versus the strength of the input noise rms amplitude is shown, the data has been obtained from computer simulations. The signal amplitude, noise amplitude, and the threshold level used at the computer simulation have been given above. The signal spikes have always had the same duration and amplitude, and the different curves show the SNR gain results obtained at different repetition frequencies of the signal spikes. Each curve have been obtained in the following way: 10000 records of the length of 16384 steps have been taken and their power spectra have been calculated via Fast Fourier Transformation. From the average of the 10000 spectra, the SNR has been calculated in the classical way. The duration of the signal spikes and the LCD spikes were 64 steps. The different filling factors of the different spike trains (due to the different repetition frequency of spikes) has been: 1/128 , 1/64 , 1/32 , 1/16 , 1/8 , 1/4 , 1/2 . The more spiky the signal, that is, the lower the repetition frequency, the greater the maximal SNR gain. In the most spiky case, that is, at filling factor 1/128 , the SNR gain goes beyond 10000. It is important to note that *no* SNR gain can be obtained for 1/2 filling factor of the spike duration, which is accordance with classical investigations, because, a sinusoidal signal is very similar to a spike train with a filling factor of 1/2 in this respect. This is probably the most important reason why researchers have been unable to improve the SNR by a stochastic resonator.

Finally we would like to summarise what do we need for an efficient improvement of the SNR:

i. Strongly nonlinear response limit.
ii. Spiky signal with low filling factor.
iii. As a stochastic resonator: an LCD.
iv. The signal spikes and the spikes of the LCD have equal duration.
v. An additive Gaussian white noise with high frequency cutoff.

We would like to emphasise, that these conditions resemble simple models of neural systems and that the improvement of the SNR reaches several orders of magnitude.

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Figure caption

Figure 1

”Naked eye demonstration” by a computer simulation. The upper curve shows the noisy signal at the input which is a mixture of a periodic spike train with an amplitude of 9 and a high-frequency Gaussian noise with an rms amplitude of 2. The lower curve shows the output of the LCD which has a threshold level of 10. With the present resolution of the picture, the lower record is just identical with the original signal spike train, so all the noise seems to be removed by the stochastic resonator. An accurate analysis shows that two types of weak noise still exist: jitter noise and rarely occurring extra spikes.

Figure 2

SNR gain versus the strength of the input noise rms amplitude. The spikes have always had the same duration and amplitude, and the different curves show the SNR gain results obtained at different repetition frequency of the spikes. Identification of the curves: at the maxima, coming from the top curve to the bottom curve, the different filling factors of the spike trains (due to different repetition frequency of spikes) has been: 1/128 , 1/64 , 1/32 , 1/16 , 1/8 , 1/4 , 1/2 . The more spiky the signal the greater the maximum of the SNR gain. It is important to note that no SNR gain can be obtained for 50% filling factor of the spike duration, which is accordance with the classical investigations.

Figure 1

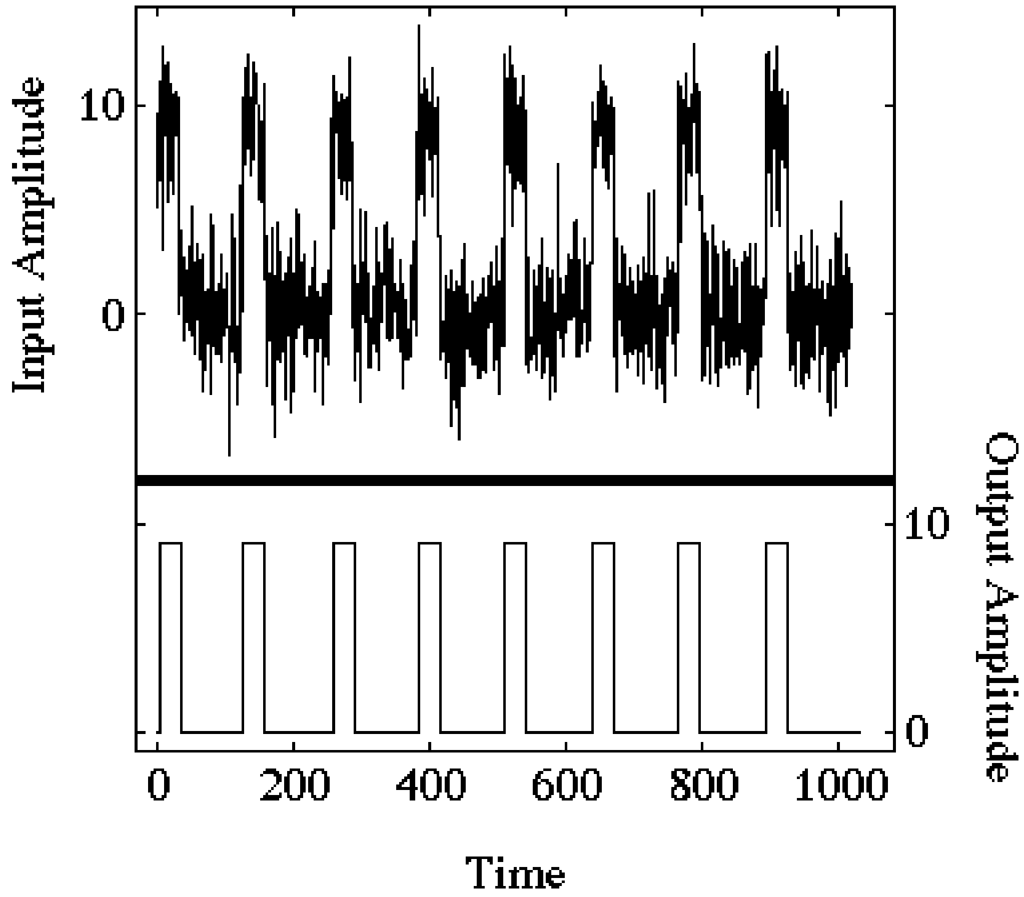
Input Amplitude
10
0
Output Amplitude
10
0
0
200
400
600
800
1000
Time

Figure 2.