

*The important thing is not to stop questioning. Curiosity has its own reason for existing. (Albert Einstein)*

## Thermal Noise Driven Heat Engines

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Electrical heat engines driven by the Johnson-Nyquist noise of resistors are introduced. They utilize Coulomb's law and the fluctuation-dissipation theorem of statistical physics that is the reverse phenomenon of heat dissipation in a resistor. No steams, gases, liquids, photons, fuel, combustion, phase transition, or exhaust/pollution are present here. In these engines, instead of heat reservoirs, cylinders, pistons and valves, resistors, capacitors and switches are the building elements. For the best performance, a large number of parallel engines must be integrated and the characteristic size of the elementary engine must be at the 10 nanometers scale. At room temperature, in the most idealistic case, a two-dimensional ensemble of engines of 25 nanometer characteristic size integrated on a 2.5x2.5 cm silicon wafer with 12 Celsius temperature difference between the warm-source and the cold-sink would produce a specific power of about 0.4 Watt. Regular and coherent (correlated-cylinder states) versions are shown and both of them can work in either four-stroke or two-stroke modes. The coherent engines have properties that correspond to coherent quantum heat engines without the presence of quantum coherence. In the idealistic case, all these engines have Carnot efficiency, which is the highest possible efficiency of any heat engine, without violating the second law of thermodynamics.

*Images: from Google Image database.*

L.B. Kish, "Thermal noise engines", *Chaos, Solitons and Fractals* **44** (2011) 114–121 ; <http://arxiv.org/abs/1009.5942>



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**We focus here on the physical viability and limits. We do not investigate if the mechanical engineering of such engine is viable with current nano/micro mechanics**

*200 years ago, at Sadi Carnot's times, mechanical engineers believed: flying based on heat engines was impossible because they imagined something like this:*



**Sadi Carnot  
1796 – 1832**



Quantum heat engines with coherence. *Some papers claim beyond-Carnot efficiency and work.*

Some others claim hat engine with a single heat reservoir. *Ongoing debate!*

a few papers;

A.E. Allahverdyan, T.M. Nieuwenhuizen, "Extraction of work from a single thermal bath in the quantum regime", Physical Rev. Lett. 85 (2000) 1799-1802.

M.O. Scully, M.S. Zubairy, G.S. Agarwal, H. Walther, Extracting work from a single heat bath via vanishing quantum coherence, Science 299 (2003) 862–864.

D. Jou, J. Casas-Vazquez, "About some current frontiers of the second law", J. Non-Equilib. Thermodyn. 29 (2004) pp. 345–357.

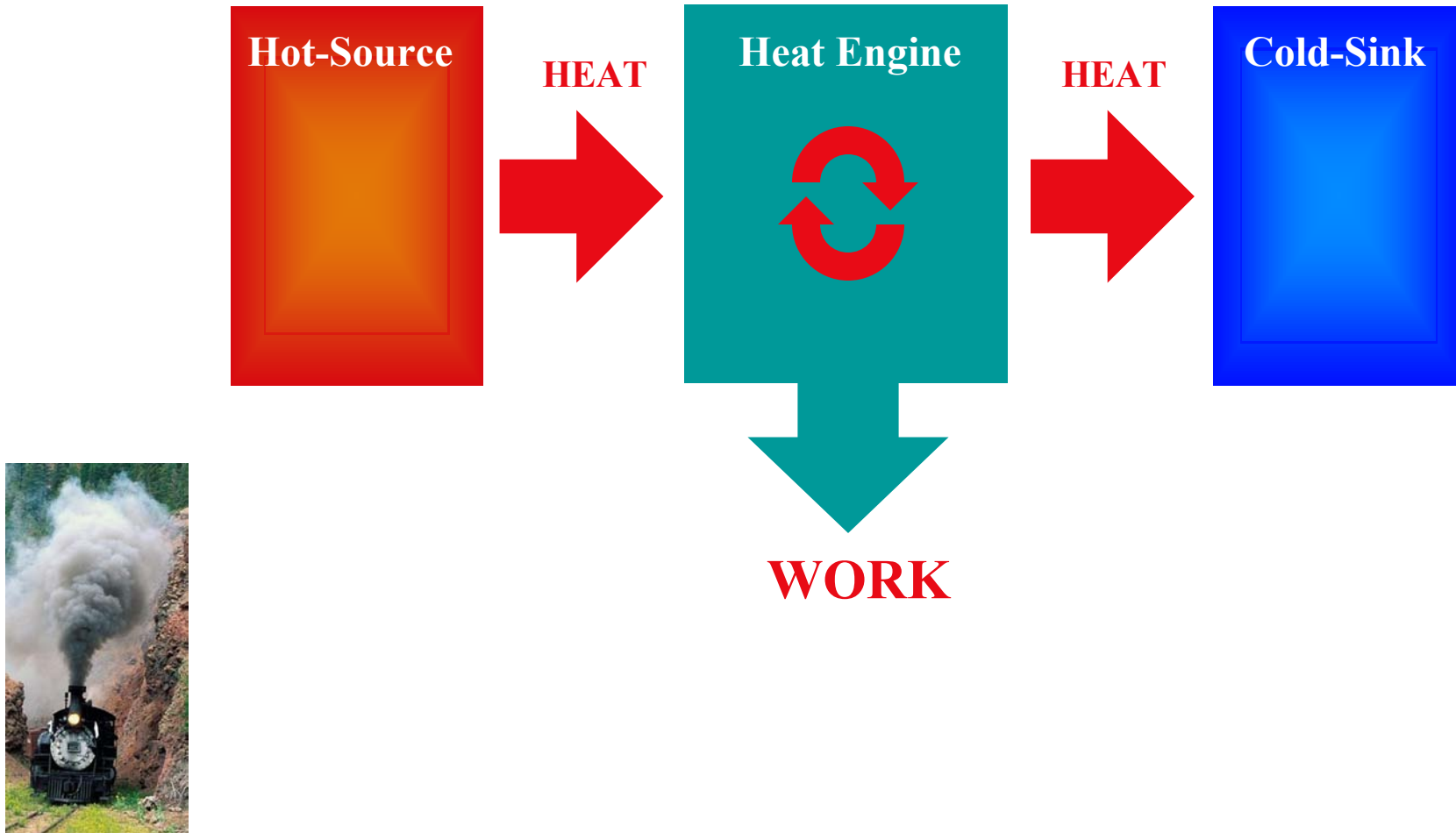
T. Zhang, L.F. Cai, P.X. Chen, C.Z. Li, "The Second Law of Thermodynamics in a Quantum Heat Engine Model", Commun. Theor. Phys. 45 (2006) 417–420.

J. Arnaud, L. Chusseau, F. Philippe, "Mechanical equivalent of quantum heat engines", Physical Rev. E 77 (2008) 061102.



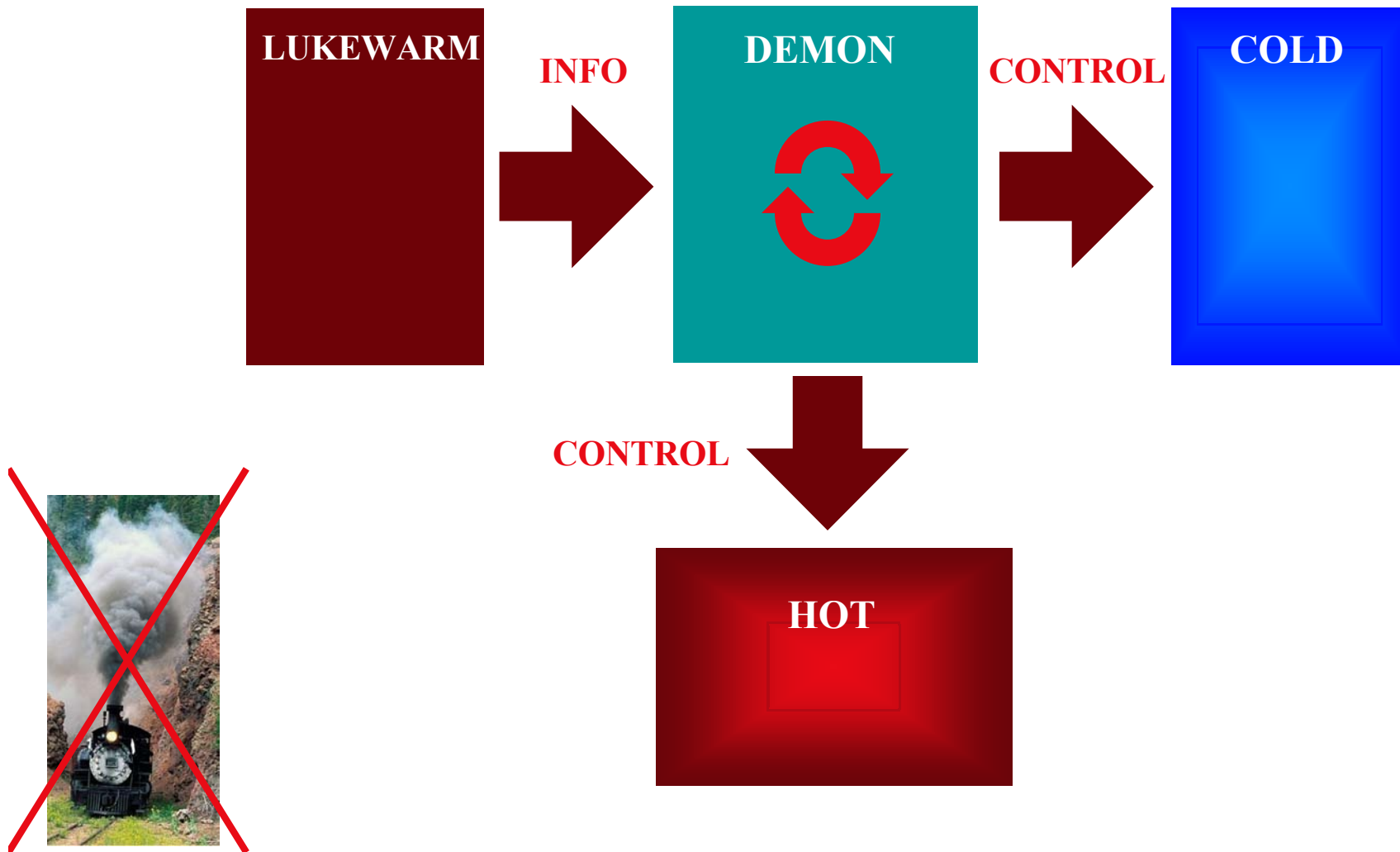
**Heat Engine: Utilizes temperature difference to produce work.**

**Result: the temperature difference will decrease unless we invest energy to keep it staying.**



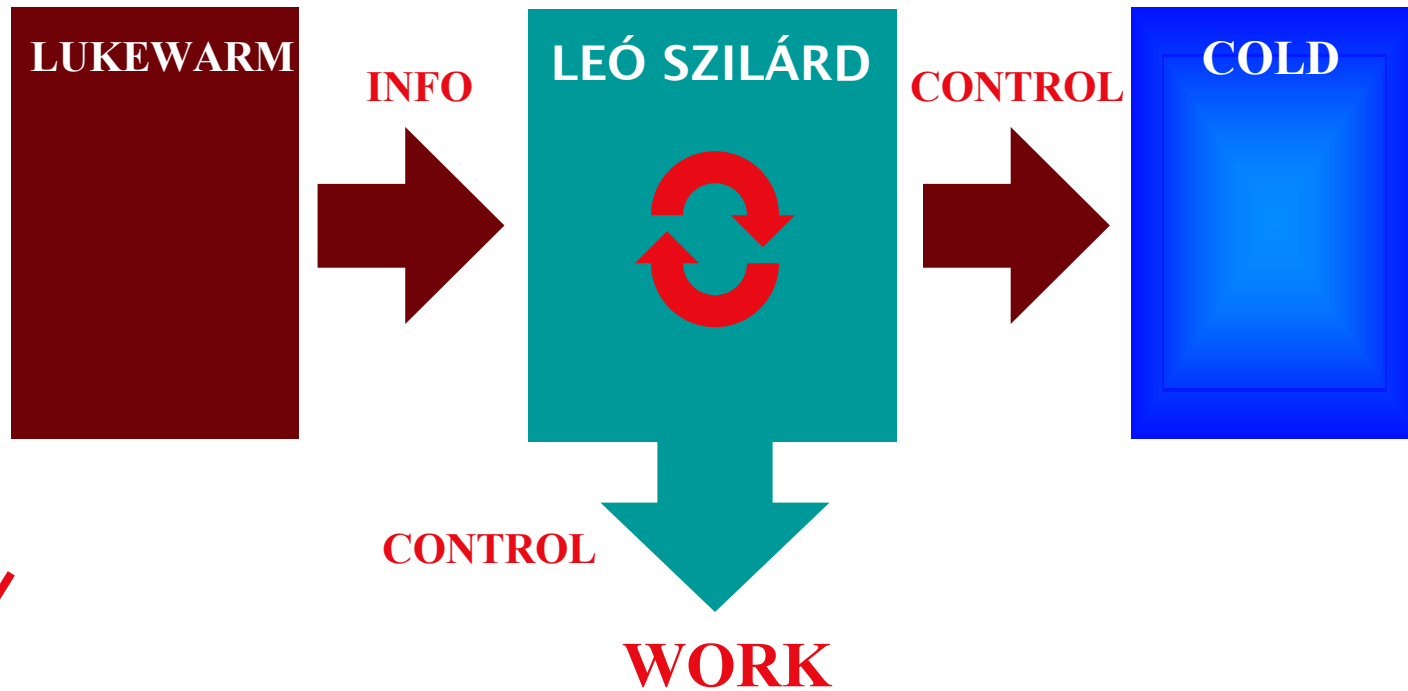
**Maxwell demon: Utilizes information to produce temperature difference.**

**Result: temperature difference.**



# Szilard engine: Utilizes information to produce work .

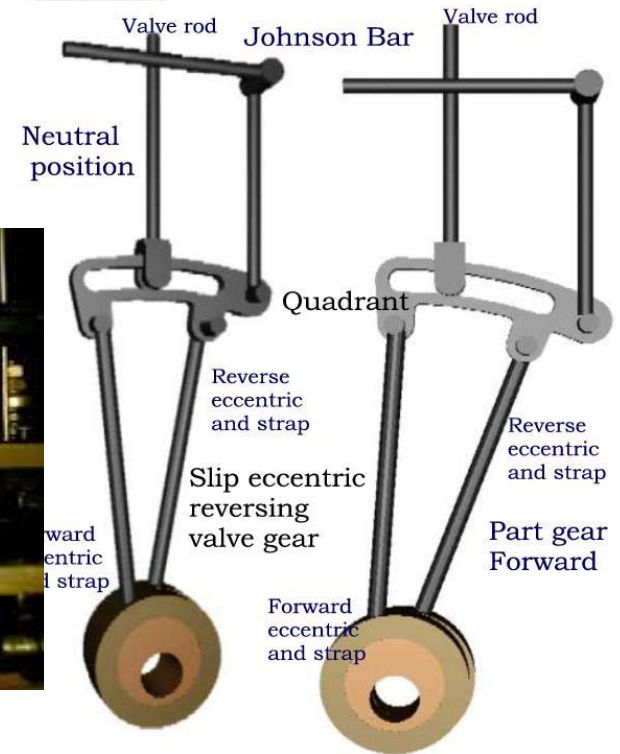
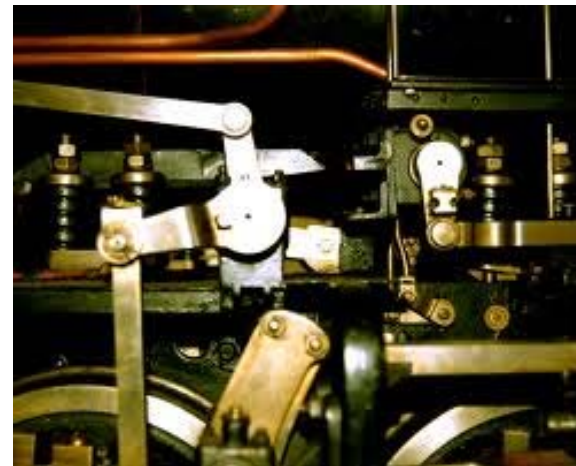
Result: work and temperature difference.



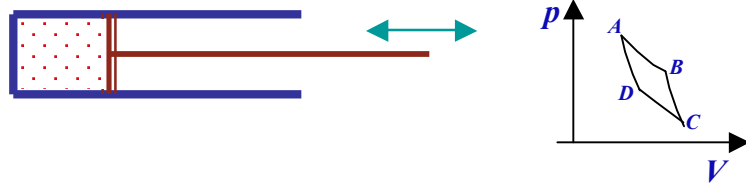


# Some of the advantages of the Johnson noise engine:

1. No steams, gases, liquids, photons, phase transitions.
2. No exhaust/pollution.
3. Voltage-controlled switches  
*(free from any mechanical motion)*  
instead of valves/levers to control the engine.



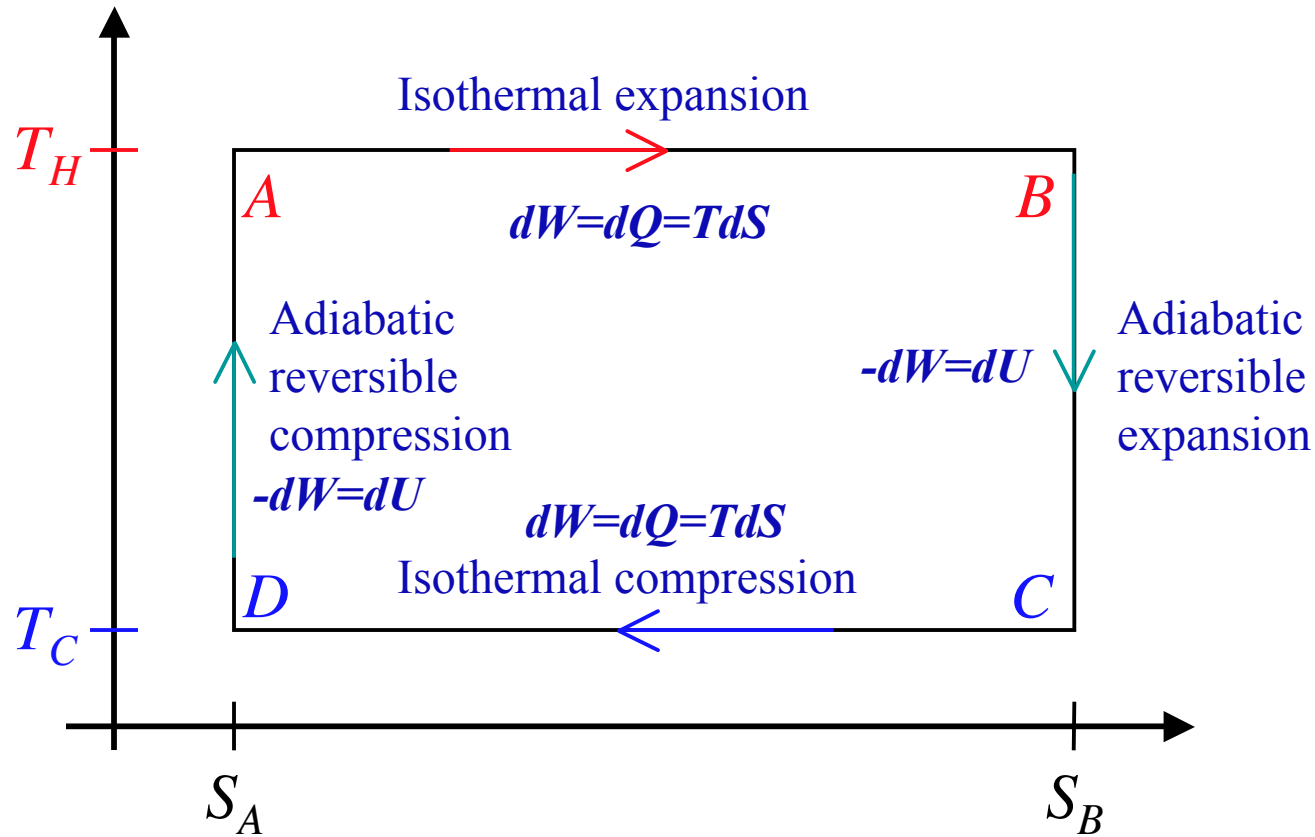
# The Carnot engine and cycle: 4 strokes



Carnot efficiency:  $\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{T_C}{T_H}$

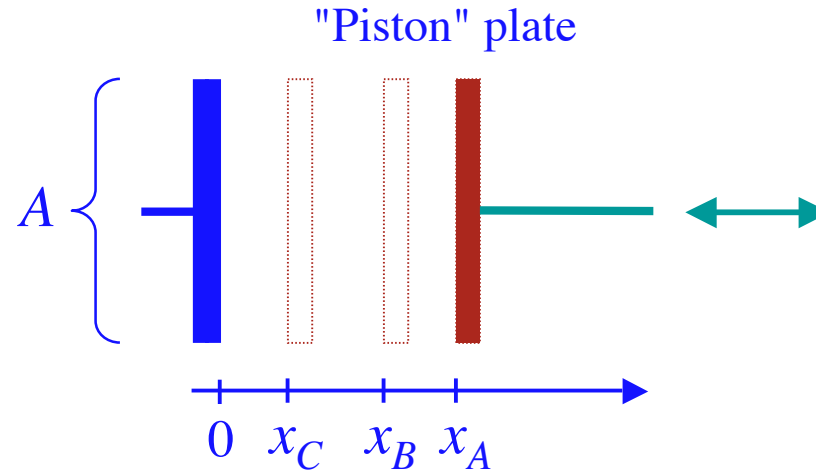


$dQ = dU + dW$





## The source of the torque: Coulomb force in a capacitor.



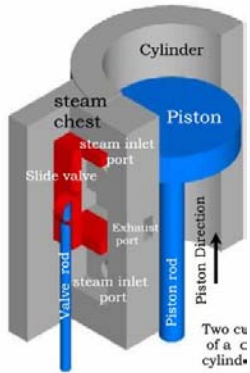
$$E_C = \frac{1}{2}kT = \frac{1}{2}C\langle U^2(t) \rangle = \frac{1}{2C}\langle Q^2(t) \rangle \quad \longrightarrow \quad \langle Q^2(t) \rangle = kTC$$

For plate-capacitors:

$$\langle F \rangle = \langle Q^2 \rangle / 2\epsilon_0 A^2 = kT / 2x$$

*Charging it by a hot or a cold resistor.*



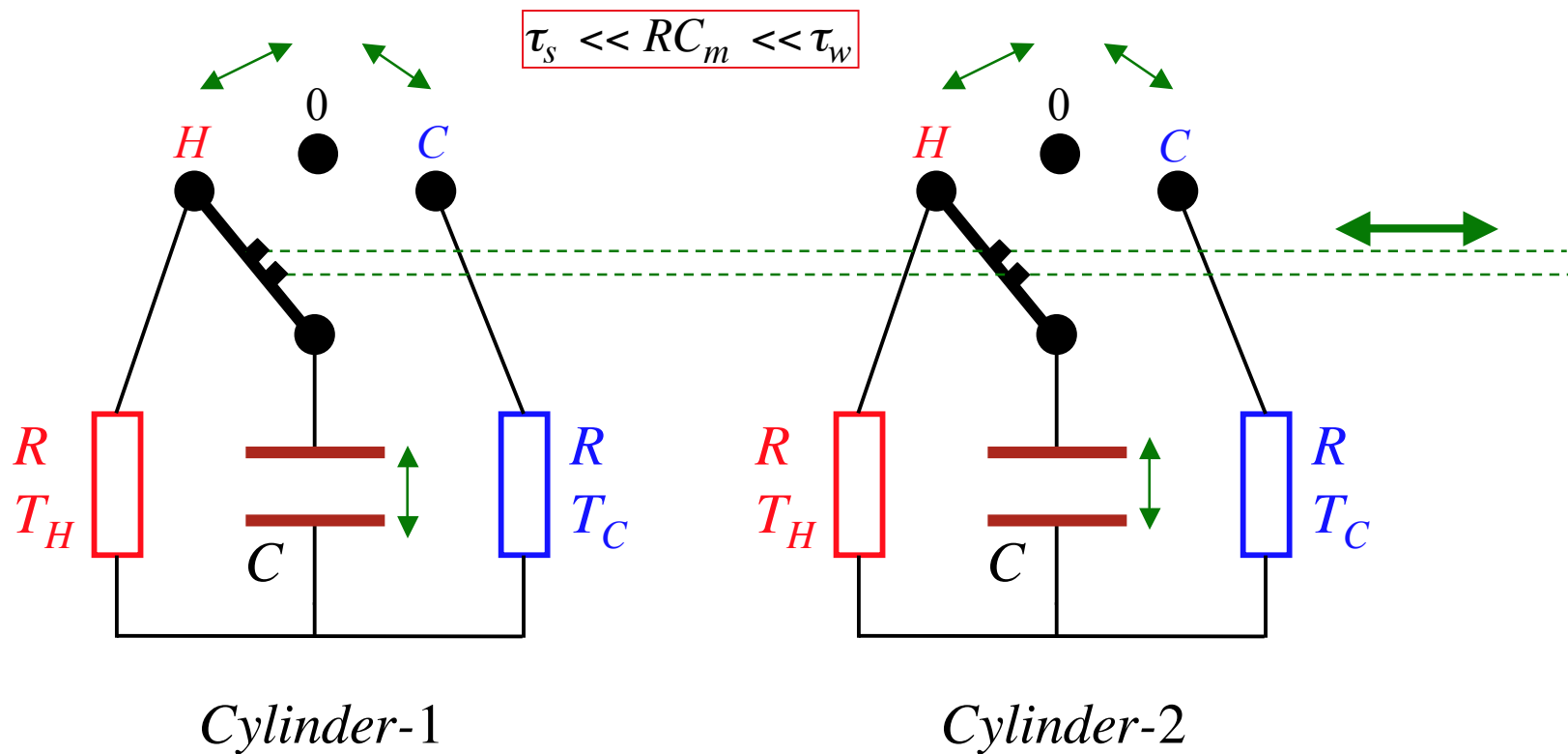


## Standard thermal noise engine

*H (Hot)*: The capacitor is connected to the "hot" resistor for an isothermal stroke.

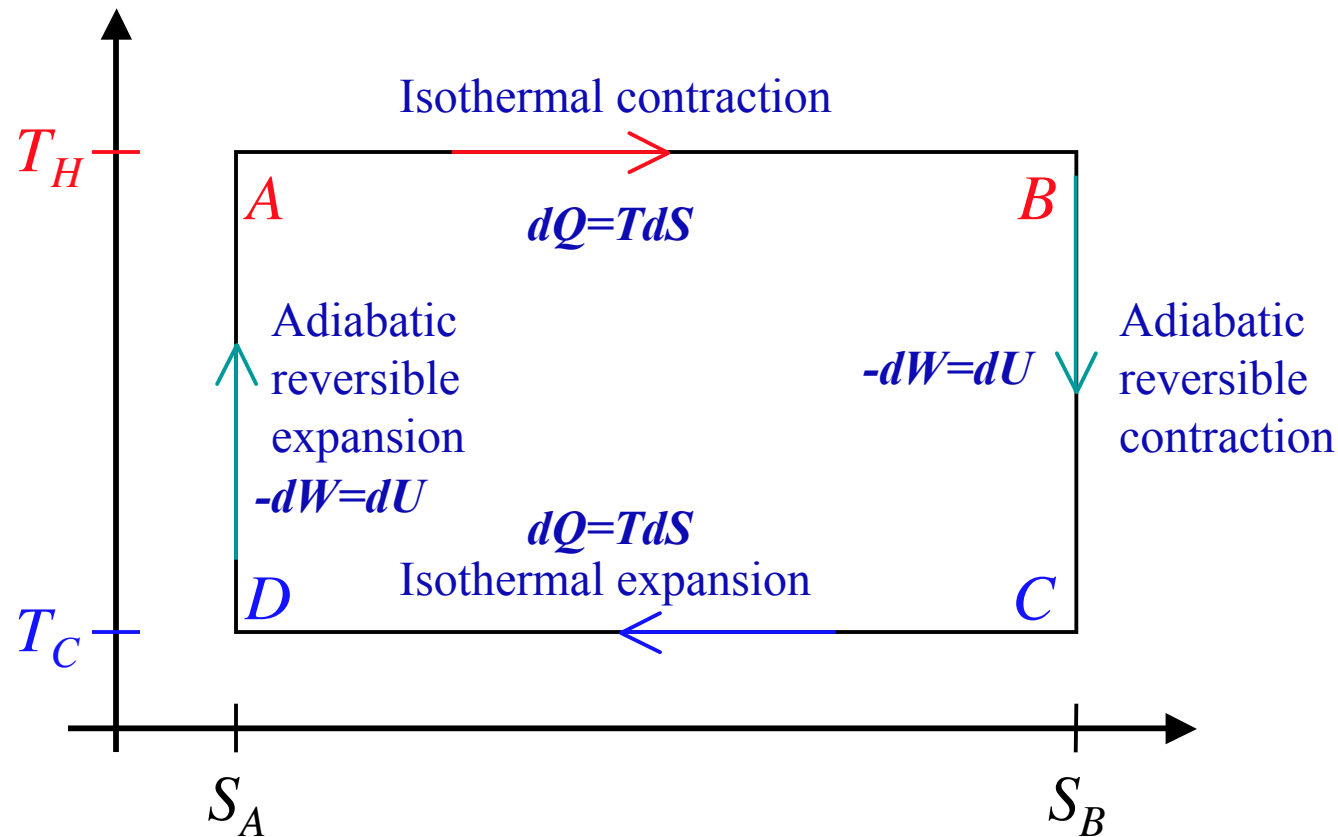
*0*: The capacitor is open-ended for adiabatic strokes.

*C (Cold)*: The capacitor is connected to the "cold" resistor an isothermal stroke.

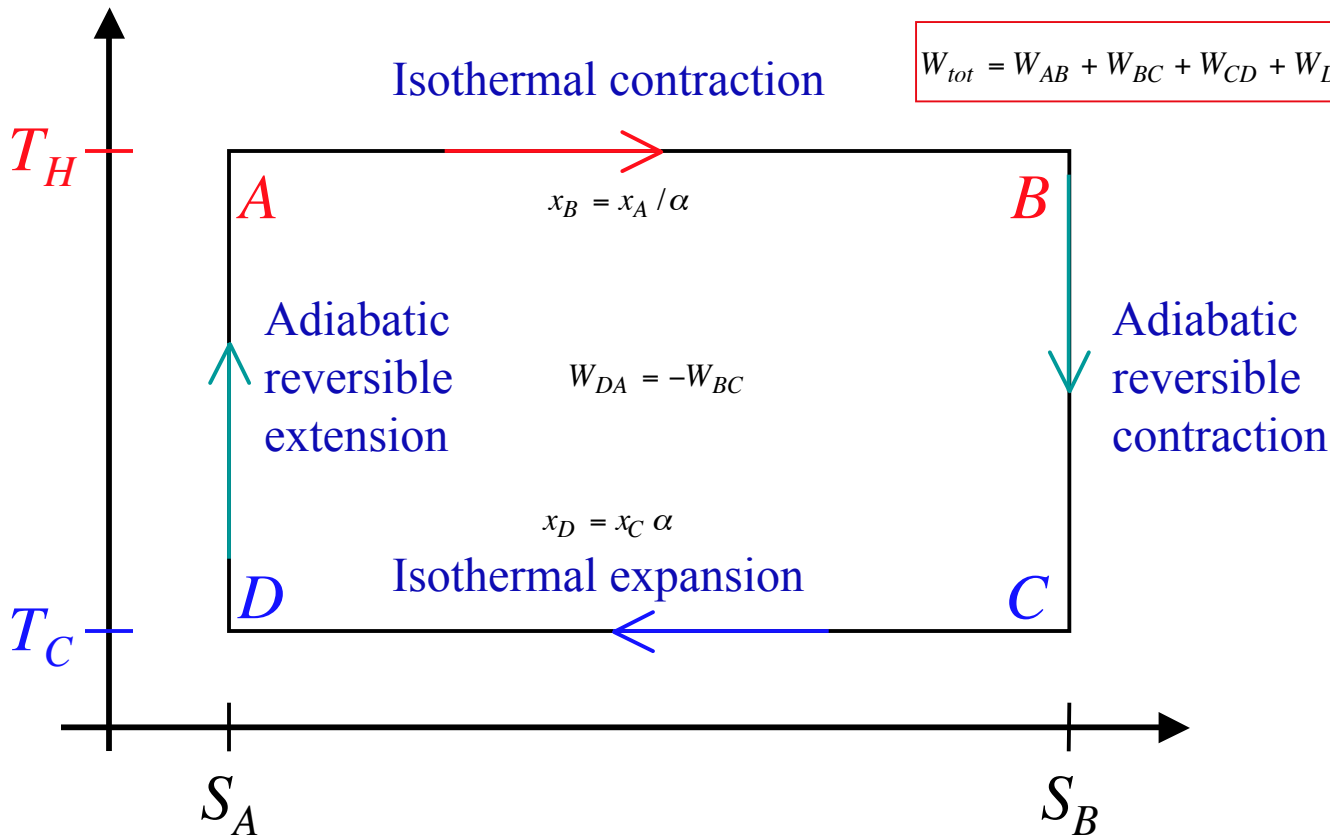


The standard 4-stroke thermal noise engine is also Carnot cycle:  
*instead of expansion: contraction; instead of compression, expansion.*

$$dQ = dU + dW$$



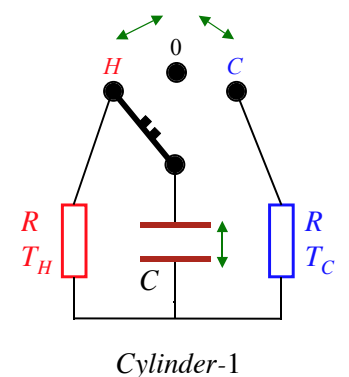
# The standard 4-stroke thermal noise engine



$$W_{tot} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{Nk}{2}(T_H - T_C) \ln \alpha$$

$$Q_H = \frac{Nk}{2} T_H \ln \alpha$$

$$\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{T_C}{T_H}$$



$$W_{AB} = T_H(S_B - S_A) = -\int_{x_A}^{x_B} N \frac{kT_H}{2x} dx = N \frac{kT_H}{2} \ln \left( \frac{x_A}{x_B} \right) = N \frac{kT_H}{2} \ln \alpha$$

$$Q_{AB} = W_{AB} = \frac{Nk}{2} T_H \ln \alpha$$

$$W_{CD} = T_C(S_D - S_C) = -\int_{x_C}^{x_D} N \frac{kT_C}{2x} dx = N \frac{kT_C}{2} \ln \left( \frac{x_D}{x_C} \right) = -N \frac{kT_C}{2} \ln \alpha$$

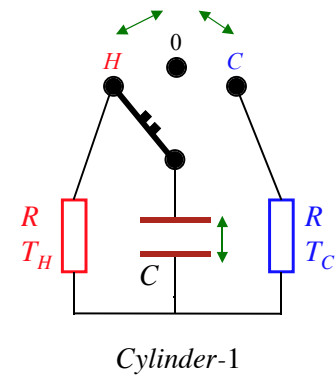
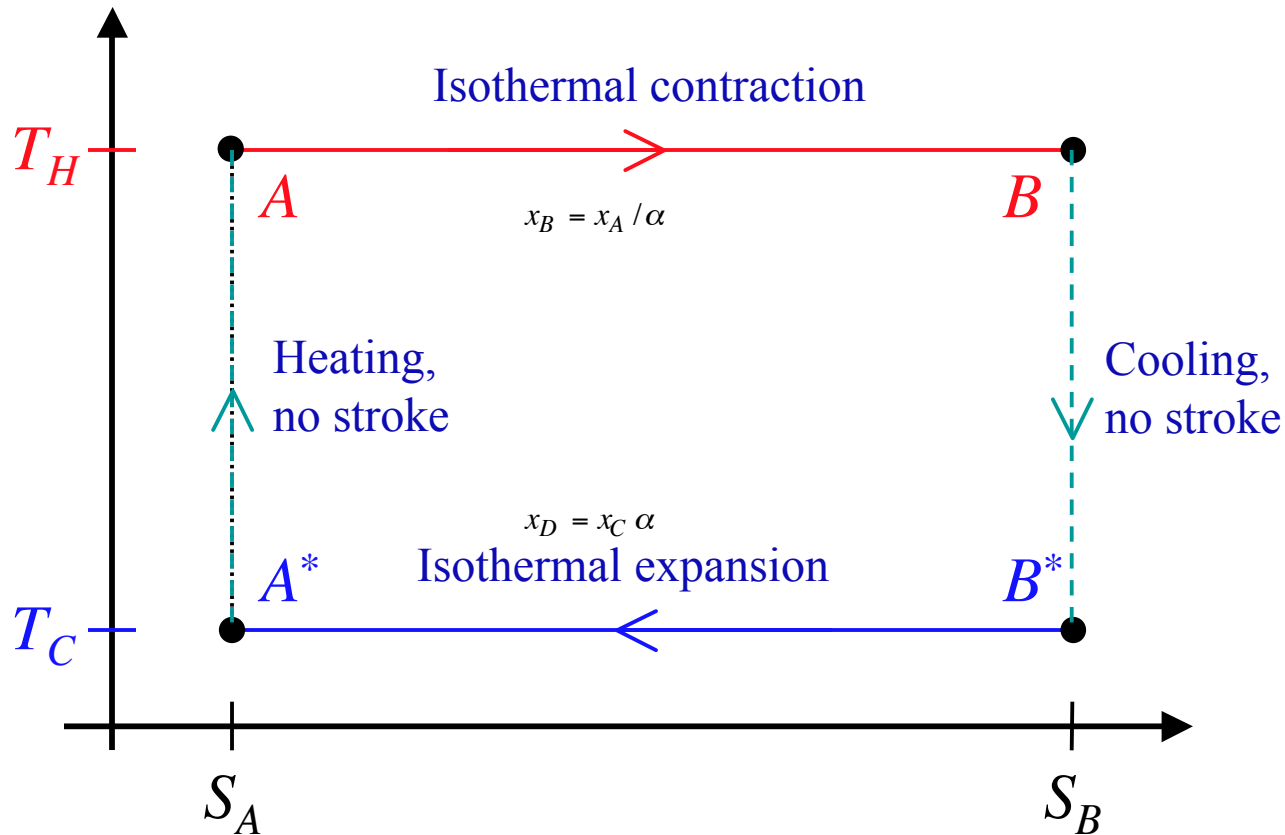
$$Q_{CD} = -W_{CD} = \frac{Nk}{2} T_C \ln \alpha$$

$$W_{BC} = -W_{DA} = Nk(T_H - T_C)/2$$



## The standard 2-stroke thermal noise engine.

Same properties. Time constants allow virtually instant cooling.  $\tau_s \ll RC_m \ll \tau_w$



$$W_{tot} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{Nk}{2}(T_H - T_C) \ln \alpha$$

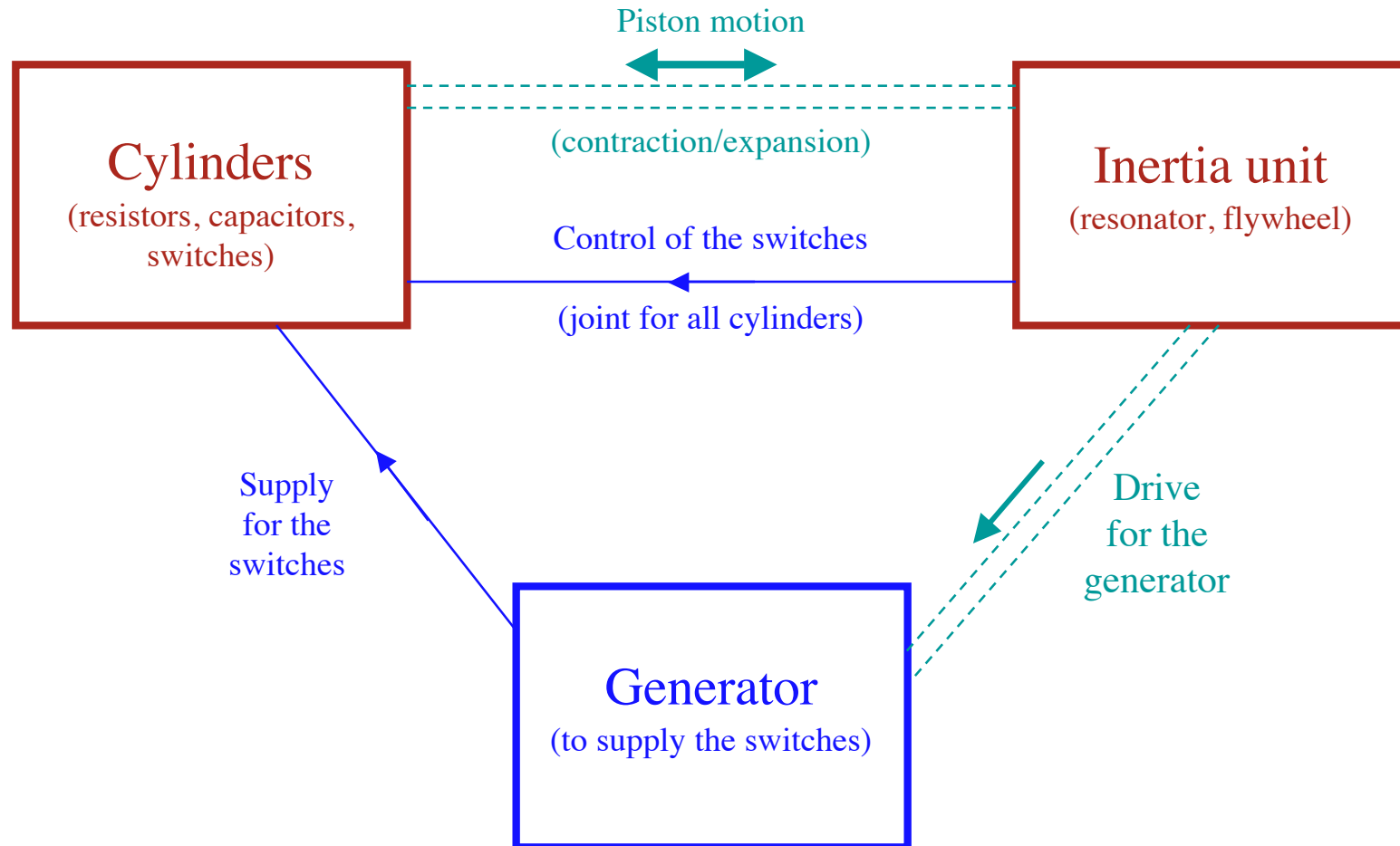
$$Q_H = \frac{Nk}{2} T_H \ln \alpha$$

$$\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{T_C}{T_H}$$

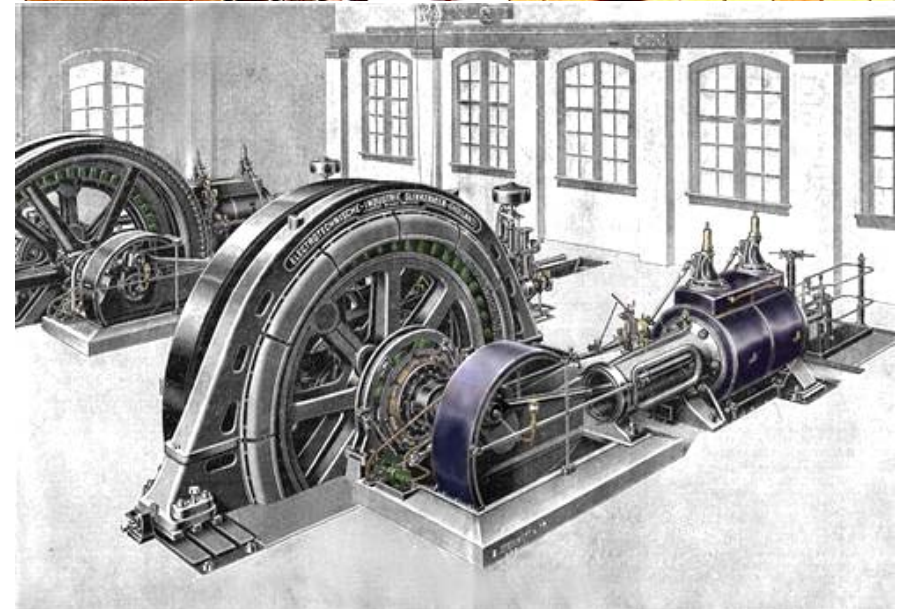
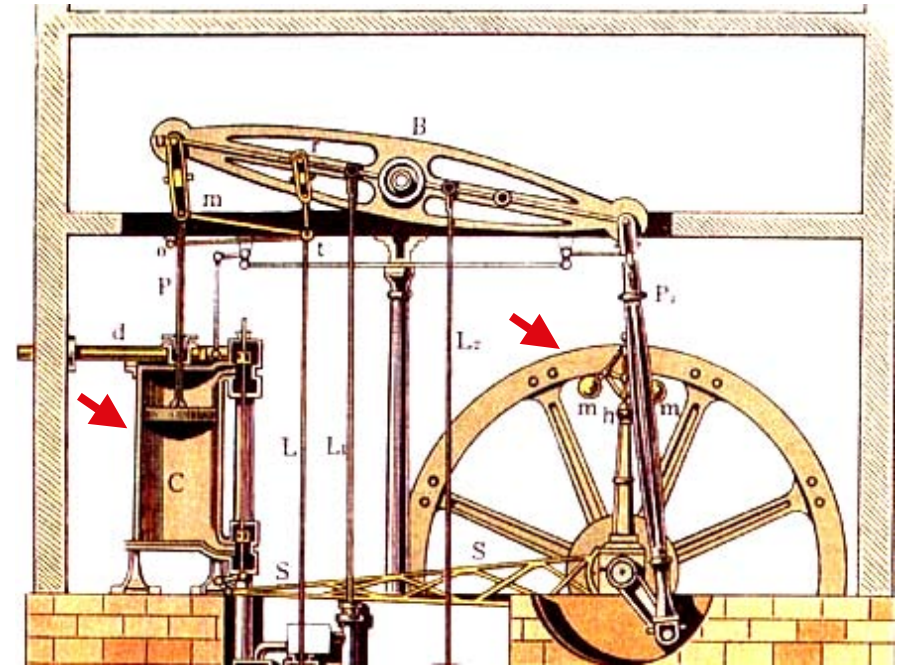
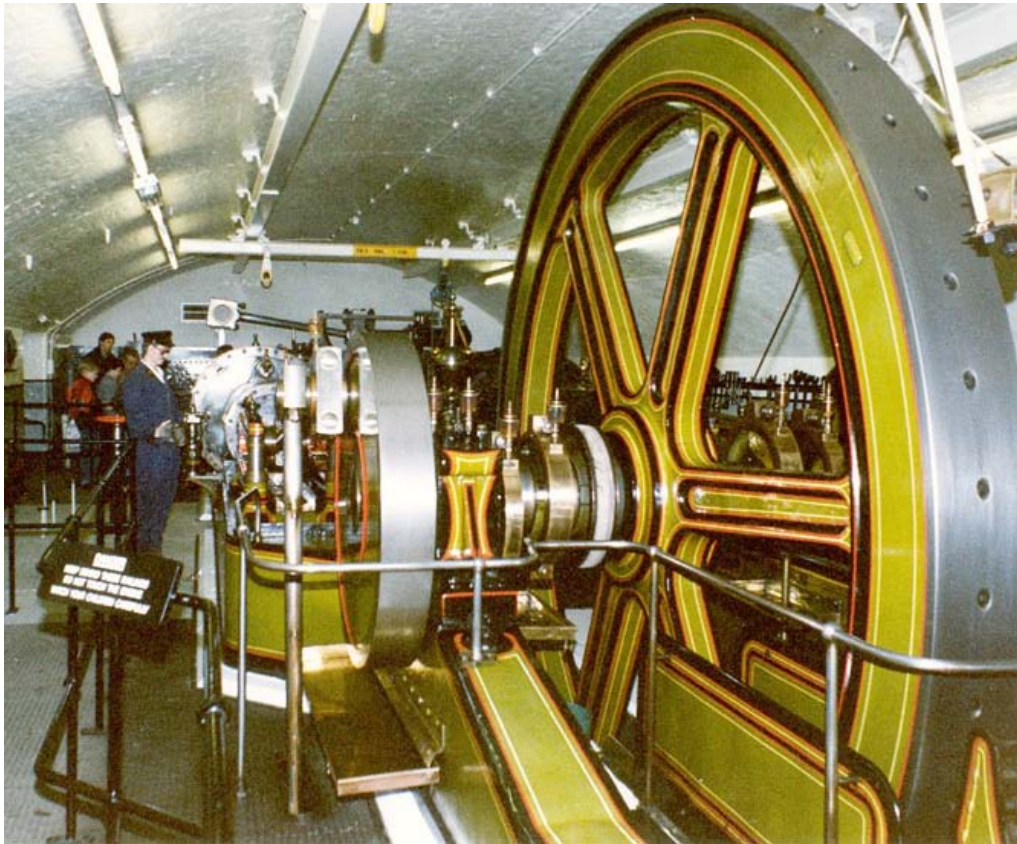




## Feasibility study. Block diagram of the generic thermal noise engine.

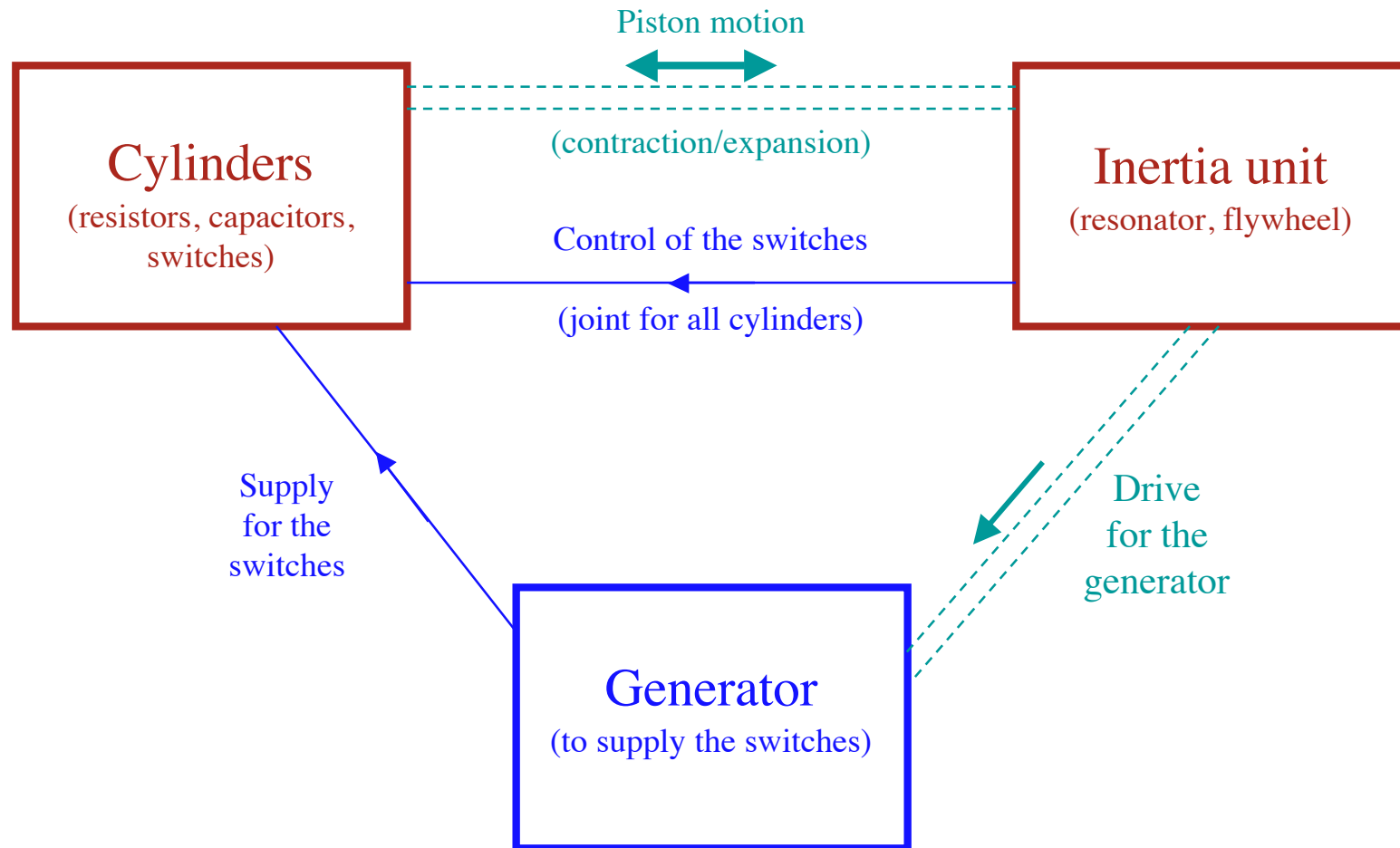


# Inertia units, flywheels as mech. energy buffers.



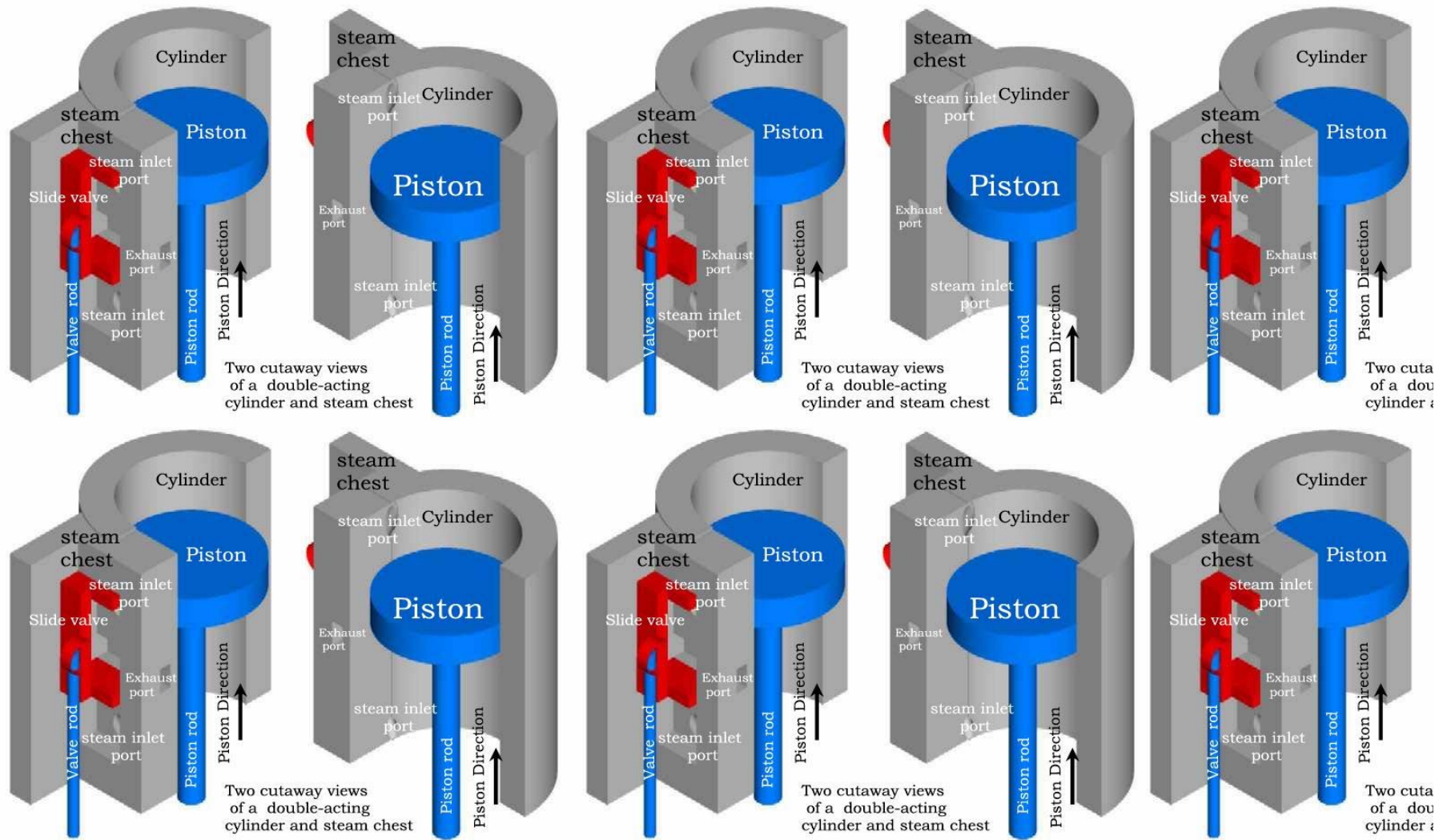
The *cylinders* are driving the *inertia unit*, which is a flywheel in usual heat engines, however, in thermal noise engines, it could rather be a *nano/micro mechanical resonator* integrated on the chip.

A small electrical energy is needed to drive the switches.





A great number  $N \gg 1$  of *cylinders* are working parallel in a synchronous way because a single cylinder offers very small work ( $< kT_H/2$ ) during a cycle. *Spatiotemporal periodicity*.  
*Note, a Maxwell demon or Szilard engine does not have this advantage.*



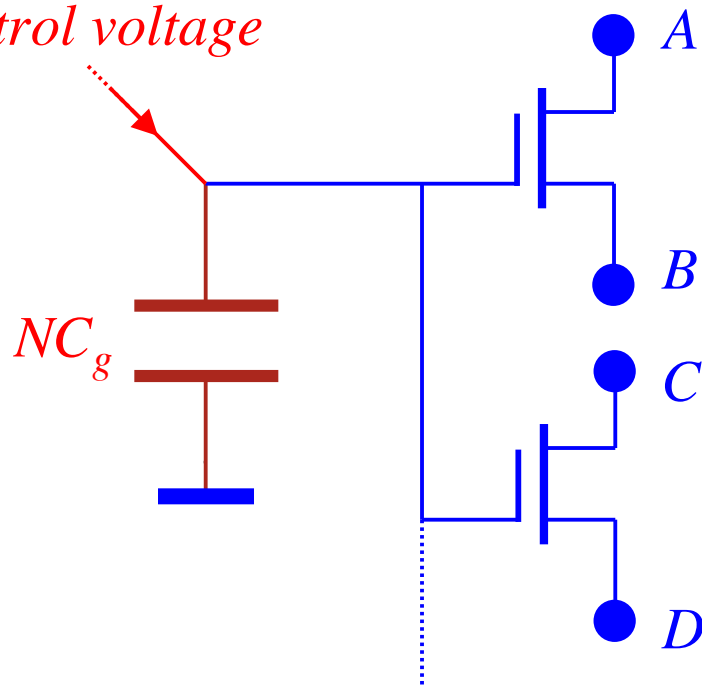
## *Spatiotemporal periodicity in driving the switches.*

To drive  $N$  *electronic switches in a synchronized way*, the *energy requirement* with the required error rate is the *same as driving a single switch* provided the voltage is enough to open the switches.

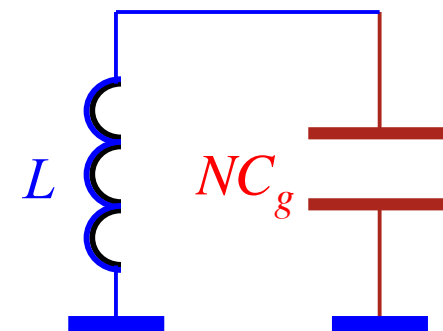
To drive the *electronic switches in a periodic way*, a resonant circuit can be used to *reduce* the energy requirement *by  $Q$ -fold*, where  $Q$  is the quality factor of the resonator.

$$E_{\min} \approx kT \ln \left( \frac{\sqrt{3}}{2} \frac{1}{\epsilon} \right)$$

*Control voltage*



1% error probability,  $4.4 kT$



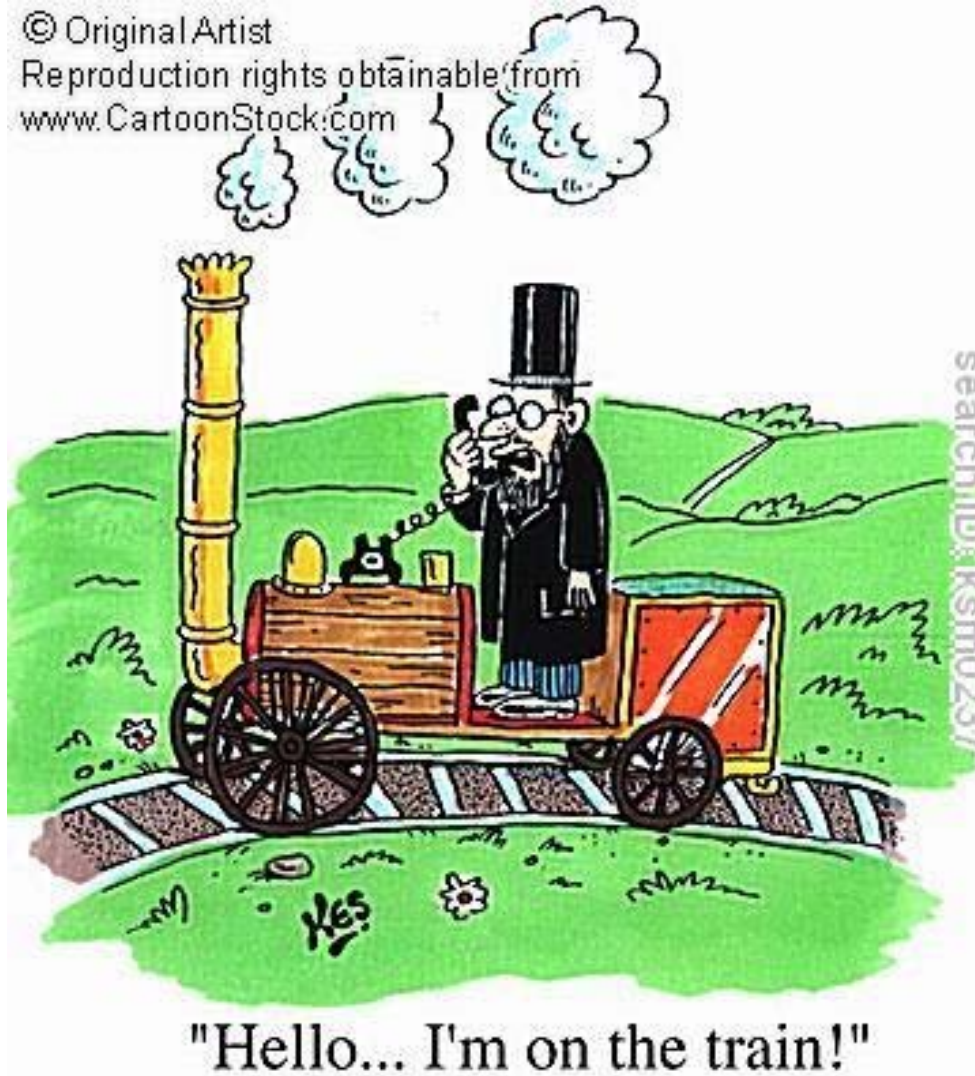
$Q$  is up 10,000 in nano-piezo-resonators

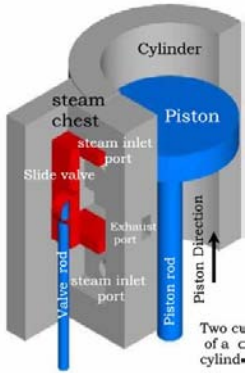
**72  $kT$ /switching cycle** low energy limit at the idealistic conditions, computers for  $10^{-25}$  error prob.  
(Kish, IEE Proc. - Circ. Dev. Syst. 151 (2004) 190-194.)





In conclusion, due to the time periodicity, we are in business even if a switching cycle requires more energy than a cylinder provides during a cycle.

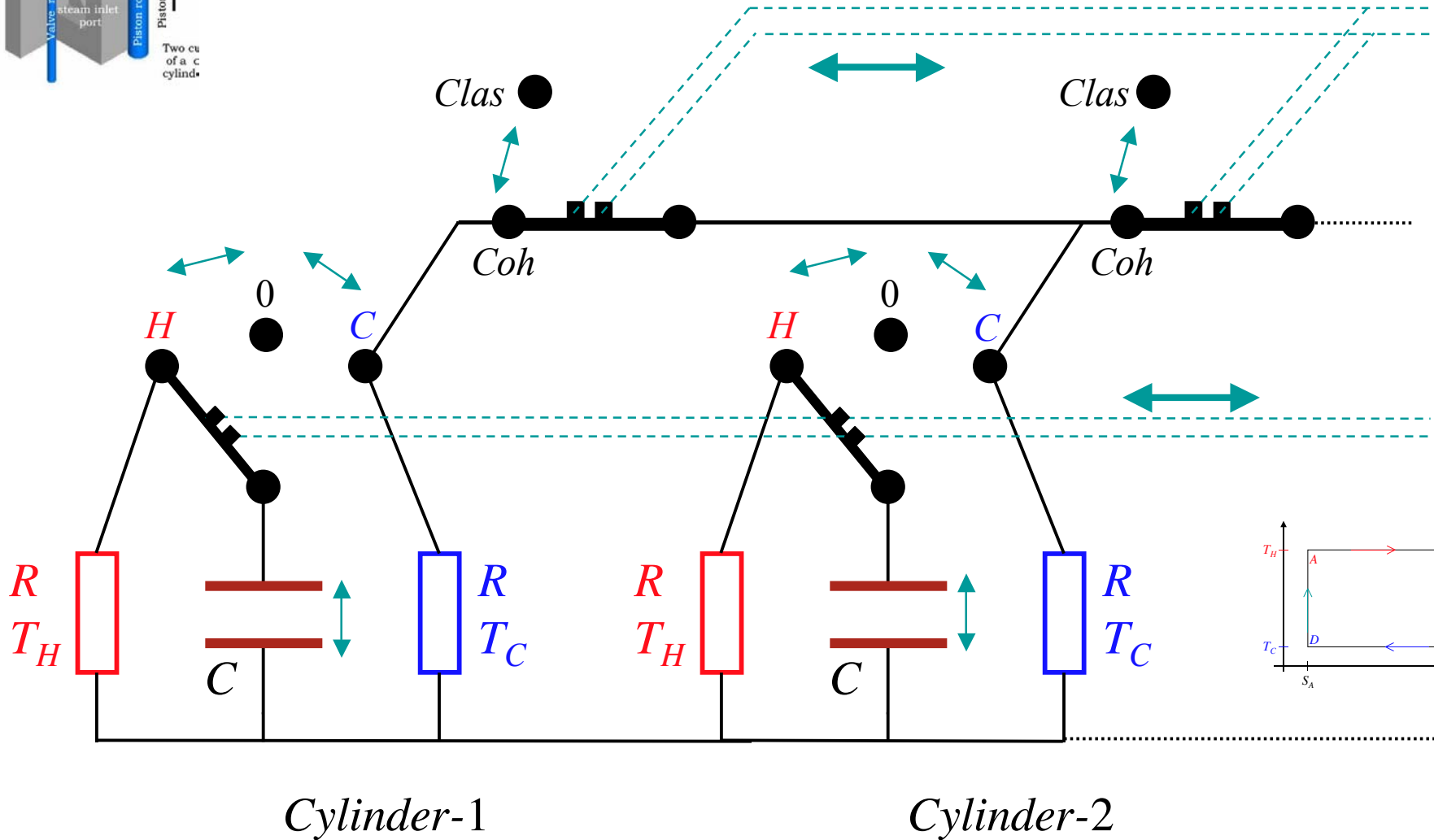




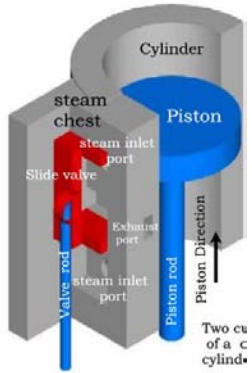
# Coherent thermal noise engine (mimicking coherent quantum heat engines)

$$W_{tot} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{Nk}{2} \left( T_H - \frac{T_C}{N} \right) \ln \alpha$$

$$\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{1}{N} \frac{T_C}{T_H} \quad !!!$$

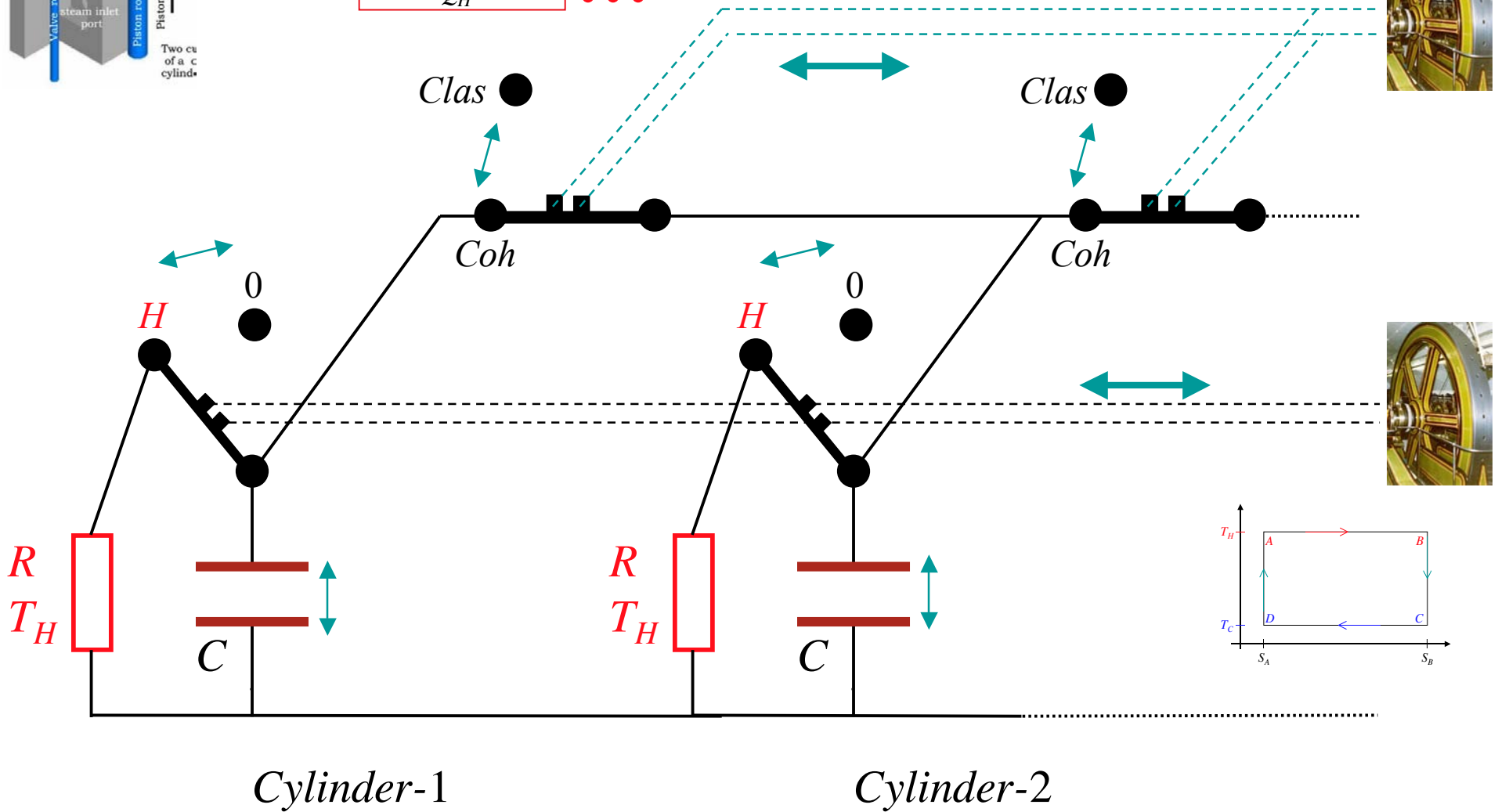


# Coherent thermal noise engine *with a single heat reservoir*



$$W_{tot} = W_{AB} + W_{BC} + W_{CD} + W_{DA} = \frac{Nk}{2} \left( T_H - \frac{T_H}{N} \right) \ln \alpha$$

$$\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{1}{N} \quad !!!$$



!!! *The Second Law of Thermodynamics* prohibits:



*Engines with single heat reservoir*

or

*Engines with greater-than-Carnot efficiency*



## *Where did we miss it?*

*Energy to make the coherence to happen?* That was the common objection against coherent quantum engines.

(Specifically, energy to drive the coherent switches? There is no other difference compared to standard engines)

# No!

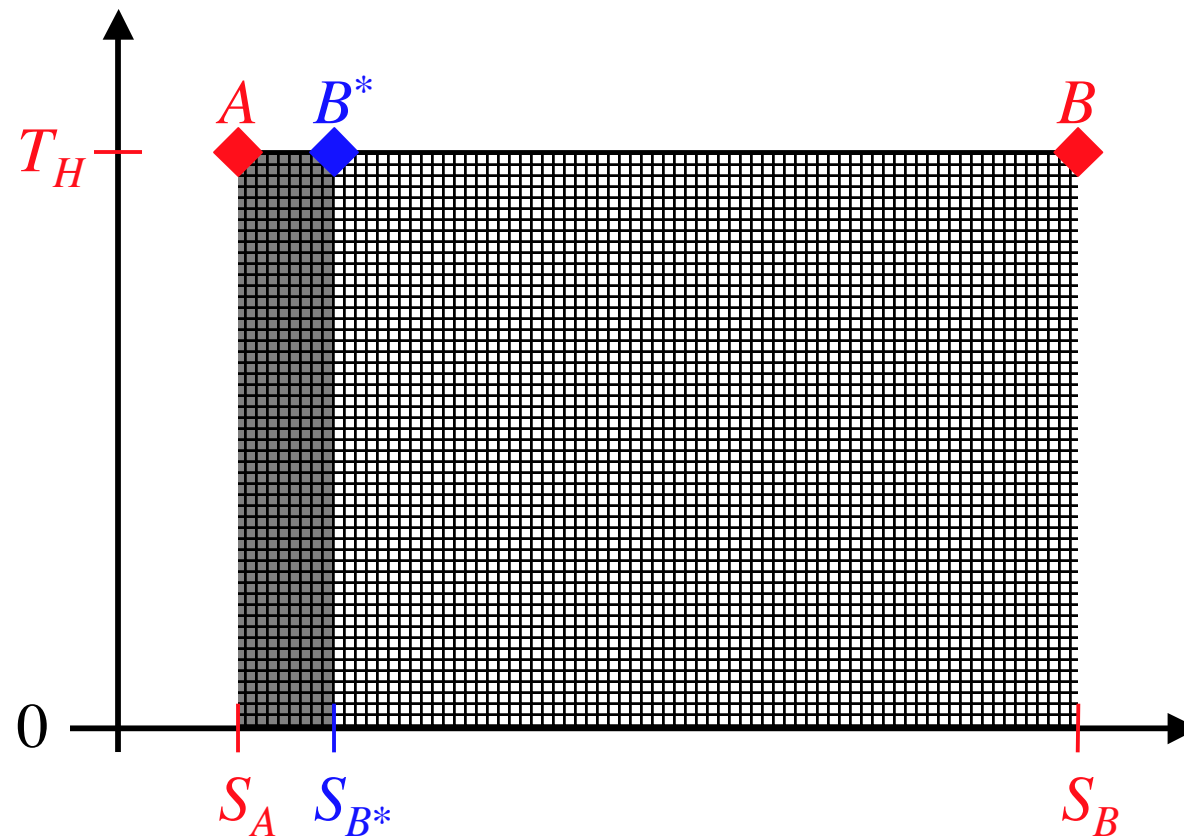




Look for the heat! *Where heat is generated, thermodynamical degrees of freedom are present.*

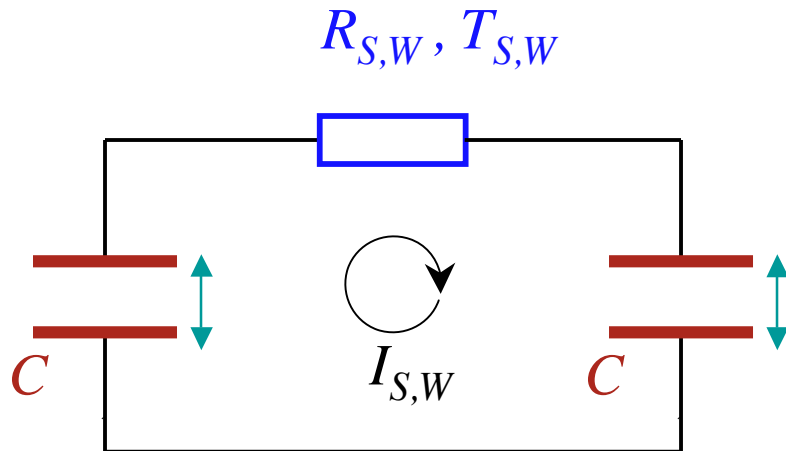
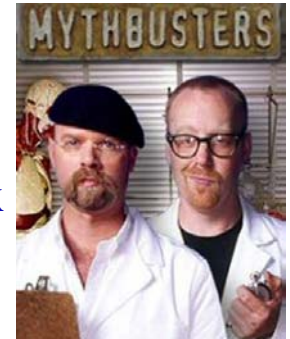
The 2-stroke coherent engine.

When the coherence is switched on, the entropy jumps from B to B\*. That is a lot of heat generation. Indeed, the  $NkT/2$  total energy, the energy in the capacitor system, jumps to  $kT/2$ .



*The catch: idealized switches and wires.* Different levels of idealization. The theory is valid for zero temperature wire and switches, which do not generate thermal noise, and that indicates a *hidden heat reservoir*.

Example:  $N=2$ . If the temperature of the wires and switches equals the cold sink temperature, here is the other half of thermal energy:



$$\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{1}{N} \frac{T_C}{T_H}$$

$$\eta_{clas} = \frac{W_{tot}}{Q_H} = 1 - \frac{1}{N}$$

$R_{S,W} C$  is the shortest time constant in the system. Fastest thermalization.

*The temperature of the medium providing the coherence is the third heat reservoir. If that is at zero Kelvin, the above results for the coherent engine are valid. Carnot efficiency is still the limit.*



**Practical considerations:** the cylinders must be *very small* to have a *large number of them* and for having a *high cycle frequency* (small resonator size) for reasonable power.



To Ohm's law the size must be greater than the mean free path of charge carriers. That means the order of 10 nanometers.

At room temperature, in the most idealistic case, a two-dimensional ensemble of engines of 25 nanometers characteristic size integrated on a 1 square inch silicon wafer with 12 Kelvin temperature difference between the warm-source and the cold-sink, and  $\alpha=0.5$  would produce a specific power of about 0.4 Watt at 3.9% efficiency (*this is the Carnot efficiency*) at 10 GHz cycle frequency.



*The important thing is not to stop questioning. Curiosity has its own reason for existing. (Albert Einstein)*

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