Cross Spectra Measure of Neural Signals and Noise

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ABSTRACT

Shannon's information rate formula does not work for wideband (aperiodic) signals with nonlinear transfer. The classical signal and noise measures used to characterize stochastic resonance do not work either because their way of distinguishing signal from noise fails. In a study published earlier, a new way of measuring and identifying noise and aperiodic (wideband) signals during strongly nonlinear transfer was introduced. The method was based on using cross-spectra between the input and the output. According to the study, in the case of linear transfer and sinusoidal signals, the method gives the same results as the classical method and in the case of aperiodic signals it gives a sensible measure. In this paper we refine the theory and present detailed simulations which validate and refine the conclusions reached in that study. The simulation results clearly confirm that the cross-spectral identifications of output signal and noise are sensible measures and we put the theory on a firm footing. As neural and ion channel signal transfer is nonlinear and aperiodic, the new method has direct applicability in biophysics and neural science.

Keywords: Stochastic Resonance, Non-linearity, Signal to Noise ratio (*SNR*), Biological information, Neural signals.

1. INTRODUCTION

Due to its relevance for biological information processing[1], the stochastic resonance (SR) effect has become one of the most promising phenomena taking place in non-linear systems driven by noisy periodic inputs[1-22]. The input of the stochastic resonators[16], has usually been excited by an additive Gaussian noise and a periodic signal with fundamental frequency f_0. The interesting effect is that, the output power spectral density shows a non-monotonic variation with respect to increasing the input noise power. That is, there exists an optimal strength of the input noise, where the system's output power density spectrum at the signal frequency f_0 has a maximal value. This effect is called SR (see Fig 1). The most important quantity of interest in SR systems is the signal to noise ratio"(*SNR*), at the input (SNR_{inp}) and at the output (SNR_{out}) of the SR system. The *SNR* is defined as:

$$SNR = \frac{P_s}{S(f_0)} \tag{1.1}$$

Where P_s is the mean squared value of the (background corrected) Fourier component of the input voltage at frequency f_0 and $S(f_0)$ is the spectrum of background noise at f_0. Of particular interest to everyone in the field is whether there exist stochastic resonance systems that can *significantly* increase the *SNR* at the output. It was shown[20] that the "old dream" of achieving

$$SNR_{out} >> SNR_{inp} \tag{1.2}$$

can be achieved, in the strongly nonlinear response limit, if we use high bandwidth noise with strong subthreshold signal which has a spiky nature (small duty cycle).

To truly evaluate the accuracy of this claim, we need a proper measure of the *SNR*, which works under *all* circumstances, not merely in the linear response case. This is because high *SNR* gains are achieved at a strongly nonlinear limit where the spectrum of the background noise is shaped by the input signal. In other words there is an interaction between signal and noise at the output and hence the signal and noise components are no longer independent. This means that we cannot measure the noise power when there is no input signal and take that as the noise component at the output. This clearly necessitates a need for a general measure for *SNR* valid in all cases. The total failure of classical suggestions for *SNR* measures becomes most obvious in the case of *wideband aperiodic signals*, which have

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been shown in [20] to include the case when significant *SNR* gain is achieved. It is important to emphasize that *all neural and ion channel signals belong to this class*. The aim of this paper is to present detailed simulations to substantiate the claim [20], that the cross spectrum method used to determine the *SNR* is indeed a valid and the most general method which works under all circumstances i.e. nonlinear limit and wideband input signals.

In section 2, we give a brief description of the SR system used in the simulations. Section 3 describes the new cross spectrum method for determining the output *SNR* and also presents simulations, which substantiate its effectiveness under a wide variety of situations. In particular, it is shown (Figs 2 and 3) that the new method agrees with the classical definition in the linear response limit. In the non-linear limit, the classical method fails while the new method still yields correct results. In section 4, the effect of a wideband input signal at the input of the SR system is considered and the simulation results indicating a large *SNR* gain at the output is presented. Section 5 gives a brief description of a new effect, termed as "Blue Shot Noise", which arises when the width of the output spike of the SR system is non-negligible. Sections 6, presents conclusions and interesting future directions.

2.DESCRIPTION OF THE LCD SYSTEM

The suitability of the cross-spectra measure for *SNR* is demonstrated using a Level Crossing Detector (LCD) setup. The LCD is a suitable candidate for study as it has a threshold like non-linearity, which is ubiquitous in most SR systems. Further extensive experimental study show that the level crossing dynamics of the Gaussian noise inherently contains the SR effect (see Fig 1). In this paper we use the LCD systems as described in [20].

Asymmetric LCD stochastic resonator:

The asymmetric system consists of an LCD of the following kind: whenever the input amplitude of the input excitation (noise and signal) crosses the positive threshold level U_t *in increasing direction*, the LCD produces a positive, short pulse with amplitude A and duration τ_0 at its output. The resulting output response of the system is a random time-sequence u(t) of uniform, positive pulses.

Symmetric LCD stochastic resonator:.

The symmetric system consists of an LCD of the following kind: whenever the input amplitude crosses the positive threshold level U_t *in increasing direction*, the LCD produces a positive, short pulse with amplitude A and duration τ_0 at its output. On the other hand, whenever the input amplitude crosses the negative threshold level $-U_t$ *in decreasing direction*, the LCD produces a negative, short pulse with amplitude -A and duration τ_0 at its output. The resulting output response of the system is a random time-sequence u(t) of uniform, positive and negative pulses with zero time average.

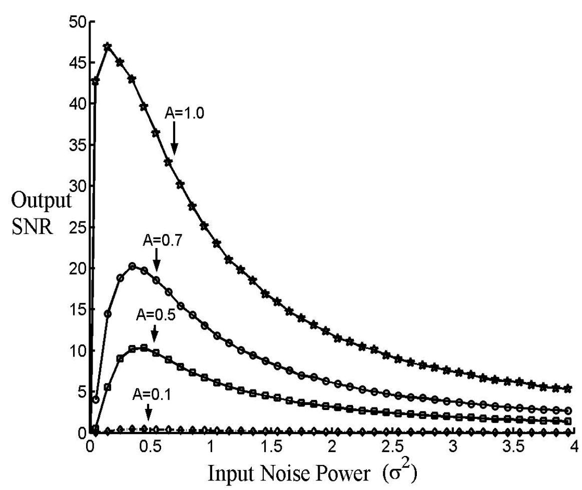

Figure 1: Demonstration of the Stochastic Resonance Effect in the Asymmetric LCD system.. The plot shows the non-monotonic variation of the Output *SNR* versus input noise for different values of the input signal amplitude A. U_t=1.

3. CROSS POWER SPECTRUM MEASURE FOR *SNR*

In general the output background noise cannot be determined by determining the output noise at no input signal. In the case of nonlinear transfer, the input signal yields extra cross modulation products with the input noise, which is an extra output noise with a strong dependence on the input signal. Furthermore, in some cases (e.g Neural signals), the input signal may not be deterministic. Hence we look for a more generally applicable definition. Intuitively the signal component in the output is that part of the output signal, which is correlated with the input signal. And it is again intuitively reasonable to define output noise as that part of the output, which is uncorrelated with the input signal. Collins and coworkers[21] have arrived at a similar conclusion and they used crosscorrelation function type measures. We will show, however, that the Collins method breaks down at nonzero phase shift between the input and the output. Our method is using cross spectra measures and it allows not only arbitrary phase shift but also to determine how large the phase shift of the stochastic resonator dynamics. With this intuition, the "generalized amplification" *K(f)* of the generalised input signal was defined in [20] as follows:

$$K(f) = \frac{S_{inp,out}(f)}{S_{inp}^{sig}(f)} \qquad (2.1)$$

$$S_{out}^{sig}(f) = S_{inp}^{sig}(f)\left|K(f)\right|^2 = \frac{\left|S_{inp,out}(f)\right|^2}{S_{inp}^{sig}(f)} \qquad (2.2)$$

where $S_{in,out}(f)$ is the cross spectrum of the input signal and the total output (output signal + output noise). Note, the exact form of the cross spectrum was not given in[20]. Here, we would like to point our that, for the phase shift invariant applicability of the cross spectral measure, the following definition should be used:

$$S_{inp,out}(f) = 2\lim_{T\to\infty}\frac{\left\langle F_{in}(f)F_{out}^{*}(f)\right\rangle}{T} \qquad (2.3)$$

where $F_{in}(f)$ is the Fourier transform of the input signal record of duration T and $F_{out}^{*}(f)$ is the complex conjugate of the Fourier transform of the corresponding output.

$S_{sig,inp}(f)$ is the power density spectrum of the input signal. Note that *K(f)* depends not only on the frequency, but also on the input signal and on the input noise. The generalised output signal component can be defined by its spectrum $S_{sig,out}(f)$. The definition of the noise power in the output signal is a straight forward consequence of the above definitions :

$$S_{out}^{noi}(f) = S_{out}^{tot}(f) - S_{out}^{sig}(f) \qquad (2.4)$$

where $S^{tot}(f)$ is the spectrum of the total output voltage. Note that the above definitions restore validity of the old definitions in the limit of small sinusoidal input signal (linear transfer and sinusoidal excitation, see Fig 2). The new definitions work at arbitrary conditions and the only pre requirement is the stationarity of the input noise, input signal and the stochastic resonator. There are fundamental differences between the classical definition of *SNR* and the above described generalized quantities:

i) In the case of deterministic signals, the above definition simplifies as follows : The signal power becomes $\left\langle\left|S_{out}(f)\right|^2\right\rangle$| and the noise power is nothing but the variance of $\left|S_{out}(f)\right|$. This leads to an intuitively satisfying view of the output signal power and noise power. This in the linear limit reduces to the classical definition.

ii) In the case of classical definitions, the signal component at the output is defined to be the square of the frequency component of the total output power spectrum at the frequency of the input signal, so that the output noise power at this frequency is subtracted. The output noise is the total output AC voltage in the case of *no signal*. Both the classical and the new definition were tested by MATLAB simulations. The input signal was a pure sinusoidal signal of amplitude 0.5 V and frequency 5 Hz and the output signal was the input signal corrupted with additive white Gaussian noise of variance 1. The threshold U_t, of the asymmetric LCD was set at 1 (see Fig 2). The Signal to Noise ratios were computed by the two methods and the theoretical value was also computed. The three values show that they all agree in the linear limit. This establishes that the new *SNR* measure gives the same value as the Classical measure in the linear limit.

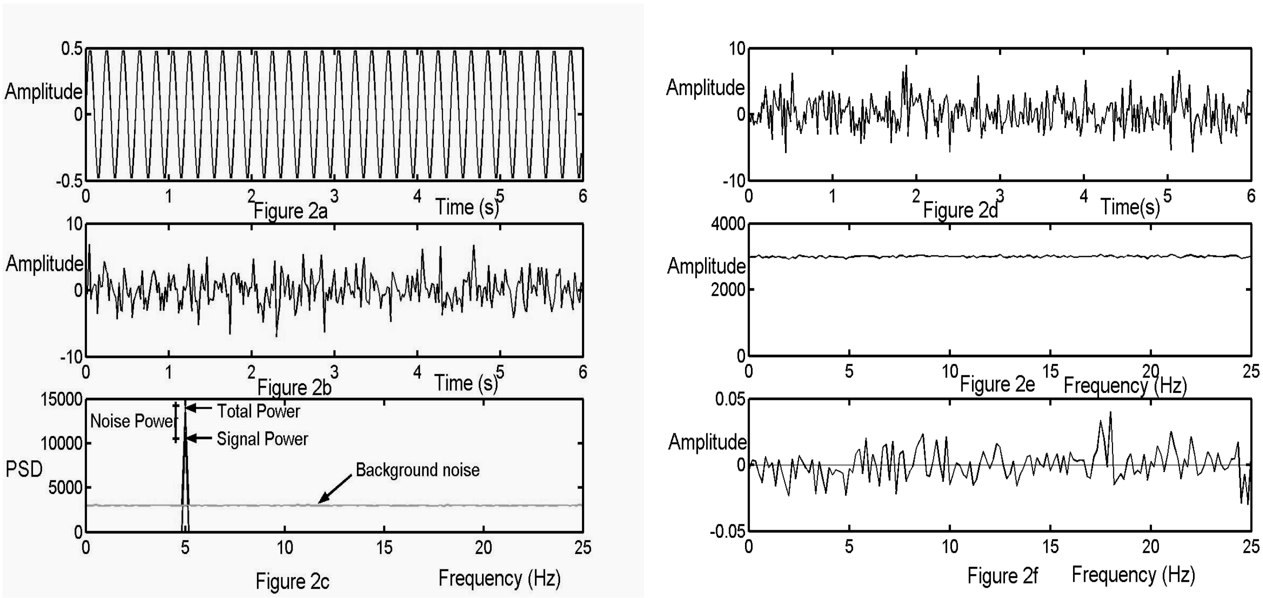

Figure 2: *SNR* determination in the linear response limit that is without passing it through a non-linear system. Fig 2(a) sinusoidal signal of amplitude 0.5 V and frequency 5 Hz. Fig 2(b) corrupted in Gaussian noise σ=1V. Fig 2(c): Total Output Power and the Signal and Noise Power components. Fig 2(d): No input signal. Fig 2(e) : Backgound Noise spectrum when the signal is absent. Fig2 (f): The difference (very negligible) of the Background noise between the without signal and with signal cases. Sampling Time = 20 ms.

Classical *SNR* : 3.7167
New Method : 3.7065
Theoretical value : 3.7500

iii) In the case of nonlinear response and periodic signals, the classical and the new method differ remarkably. The classical measure fails even for very strong periodic input signals because then the output noise can be suppressed due to overloading the resonator by the signal. The input signal was a pure sinus of strong amplitude (i.e. comparable to the noise variance), corrupted by an additive white Gaussian noise of variance 1. This signal was passed through the Asymmetric LCD described in Section 2 (U_t.=1) The background output spectrum is compared to the background spectrum when only the input noise is present. It is clear that the presence of the signal definitely has an effect on the shape of the background noise spectrum (see Fig 3). The signal to noise ratio is now computed by both the classical and new methods. There is a significant difference between the two. The output noise given by the classical method is higher. Now, the background noise spectrum must be continuous with frequency. Hence one can compute the noise power at the signal frequency from the plot of the output power density spectrum using this argument. This value agrees with the value given by the new method as shown in Table 1, clearly showing the inadequacy of the classical method for the nonlinear case. These results unambigously confirm the validity and effectiveness of the new method.

Signal Amplitude	Classical *SNR*	New Method	*SNR* by continuity argument
A=0.5	11.1490	12.5911	12.6090
A=1.0	30.9393	44.7374	44.7333
A=1.5	31.8721	60.4989	60.5106

Table 1: Comparison of *SNR* obtained by Classical, New and Continuity methods at nonlinear limit

However, the value given by the continuity argument can be unreliable because the height of the noise power is determined manually from the plot, where the area below the spectral spike has to be determined for that. Whereas the method using cross spectra yields the same result mathematically and hence has not only a better reliability but also can be employed in a straightforward mathematical formulation.

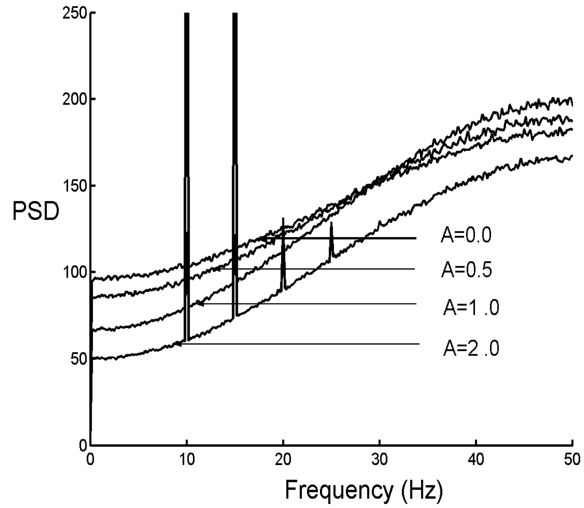

Figure 3: Plot of the Output Background Noise for various amplitudes of the input sinusoidal signal of frequency 5 Hz. Clearly the background noise changes significantly in the nonlinear limits.

iv) As we mentioned above, a measure for the output signal power using correlation coefficient was proposed [21] by Collins and coworkers. Though the Collins method works nicely in systems with no phase shift between the input and the output, it fails in the case of phase shift. But this has the disadvantage of not being robust to phase errors. For example if the input is a sinusoid whose phase is unknown, then a 90 degree phase shift between the actual and assumed phase will result in the correlation coefficient being zero. Our cross spectral measure does not suffer this drawback as it is shown in the simulation in Fig 4. The input is a sinusoidal signal of unknown phase and the stochastic resonator shifts the phase by 90^o. Still the output signal does not go to zero. The imaginary part of the cross spectrum can be used to compute the phase difference of the output signal with respect to the original signal.

v) Wideband aperiodic signal with phase shift: It is obvious from the above results and considerations that, presently, the only method able to provide usable results is the cross spectral method. That means, biophysical applications have no other choice, so far, than to use cross spectra.

vi) In Fig 5, further comparisons between the *SNR* determined by the classical and the cross spectral methods is shown. Here the input signal was a sinusoid corrupted by Gaussian noise. The simulations were carried out for different values of the amplitude of the sinusoid. Clearly at the non linear limit (higher signal amplitudes), the classical method is inadequate. The results shown above (Figs 2,3 and 4) clearly show that the cross spectrum method is a consistent measure at all ranges. The plot in Fig 5, gives us an estimate of the error made by the classical method in the strongly nonlinear limits and also high values of input noise. Thus all the simulations presented so far point out the fact the new *SNR* measure is beyond doubt both a correct and a convenient one to use under a wide variety of circumstances of practical importance.

4. STOCHASTIC RESONANCE AND SIGNAL TO NOISE RATIO ENHANCEMENT

One of the simplest possible systems for getting a *SNR* enhancement is the asymmetric LCD which was described in Section 2, which is driven by the sum of a Gaussian white input noise and a sequence of square input signal pulses. To represent a wide band input signal, it is assumed that the initial times of the pulses are generated by a Poissonian process, with a rate v_{sig}. However any stationary time sequence would give similar results as far as the improvement of *SNR* is concerned. The present signal is a good representation of a real pulse frequency modulated (PFM) signal. As we only want to demonstrate that this system is able to improve the *SNR* significantly, for the sake of simplicity we choose the output characteristics that way, so they can make the calculations easy. Therefore it is assumed that the square input signal pulses have the same duration τ_o and size A both at the input and at the output of the LCD. Moreover the rms amplitudes of the input noise and A are assumed to satisfy the following relations with the threshold level U_t.

$$\sigma << U_t \\ U_t - \sigma < A \tag{4.1}$$

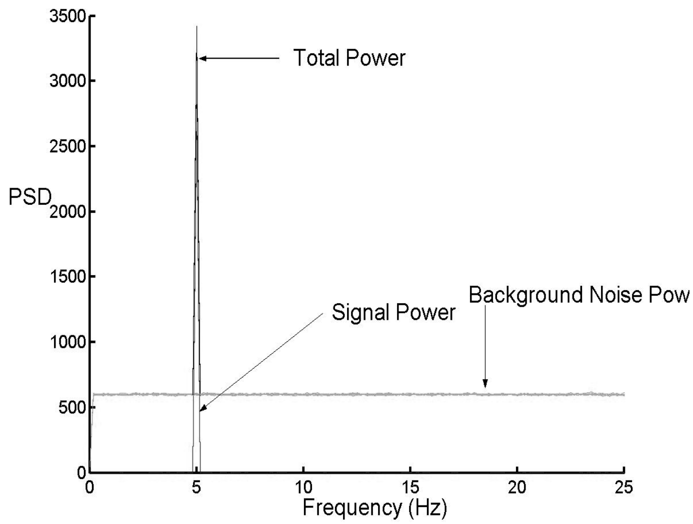

Figure 4: Cross spectrum measure in the case of unknown phase of the input signal. Here the input is a sinusoid shifted in phase by 90 degrees and frequency 5 Hz. This is correlated with a sinus of the same frequency with zero phase shift. The signal component of the output signal as predicted by the cross spectral measure is clearly robust to phase errors.

And the input noise has a very high cut off frequency f_c as compared to the reciprocal length $1/\tau_0$ of the pulses:

$$1/\tau_0 << f_c \tag{4.2}$$

Under these conditions it is observed from the simulations that a significant *SNR* enhancement occurs at low repetition rates. It is remarkable to note that close to the maximum of possible repetition rate , there is no gain but significant loss of *SNR*. As far as biological applications are concerned, the consequences of this fact has to be analysed in terms of information theory and real strong reduction of the *SNR* at the input, but less strong *SNR* reduction at the output. It is due to the jitter noise component, which is proportional to the rate , similarly to the signal. At very small v_{sig}, *SNR* gain is saturating due to the classical output noise which is independent of v_{sig}. Note the saturation limit of *SNR* gain can be arbitrarily large by choosing proper values of A and the threshold level. The theory behind this is described in detail in [20]. The simulations in Fig 6, confirm the theory and the validity of the measure for *SNR* introduced.

5. BLUE NOISE

An interesting phenomenon occurs when the width of the spike produced at the output of the Asymmetric LCD is increased. The spectrum of the output background noise shows a peak. This peak is quite close to $1/\tau$, where τ is the width of the spike generated at the output of the LCD. Thus actually at low frequencies the output noise power increases with frequency. As such a behavior is opposite to “Red Noise” spectrum, this effect can be termed as “Blue Noise”. This effect occurs when the input noise is quite strong. Hence the probability of the noise crossing the threshold is very high and the memory of the system (which comes into picture because of non-negligible spike width) prevents the LCD from firing until the current spike is over. This memory forces some kind of periodicity in the output noise. A more detailed explanation of this effect can be found in [22]. Figure 6, shows the output background noise spectrum for different values of the width of the output spike.

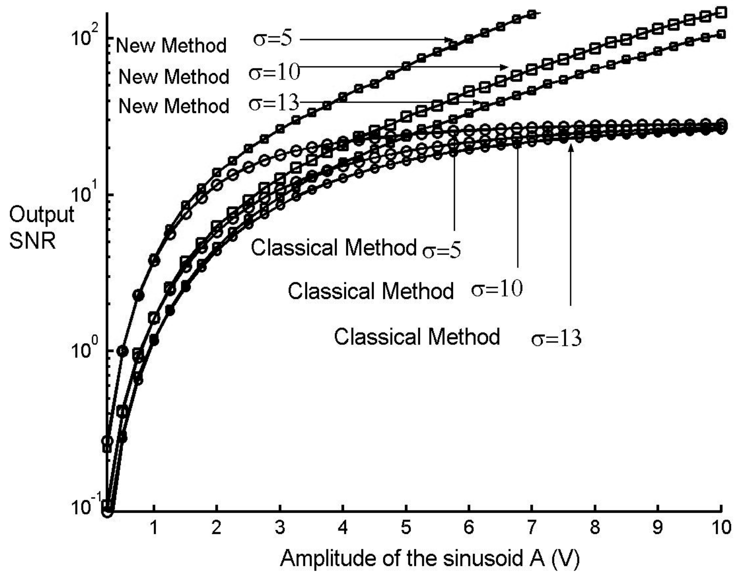

Figure 5: Comparison of *SNR* determined by classical and new method as the amplitude of the input sinusoidal signal varies. The comparisons are presented at different values of the input noise power. In the linear limit there is a close agreement and in the nonlinear limit the error made by the classical method is quite substantial. The error of the classical method reaches one order of magnitude.

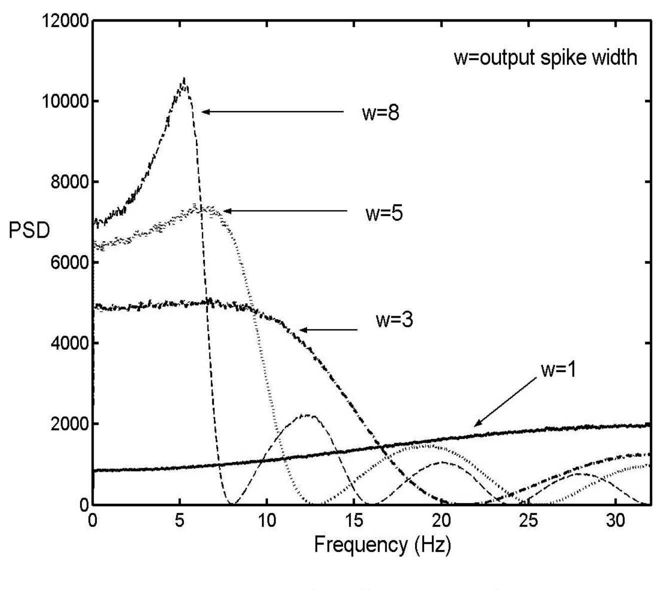

Figure 6: Blue Noise Effect. The output background noise spectrum for different values of the output spike width of the LCD. When the spike width w, is higher, the noise power increases upto a frequency and then comes down.

6. CONCLUSIONS

This paper has shown simulations results to show the efficiency of cross spectra measure for signal and noise in the case aperiodic spiky and other wideband signals in the strongly nonlinear limit. The results show that the cross-spectral identifications of output signal and noise are sensible measures and that they work for arbitrary signals and noise, for both the linear and nonlinear cases. As the neural and ion channel signal transfers are nonlinear and aperiodic, the new method has direct applicability in biophysics and neural science.

7. ACKNOWLEDGEMENTS

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