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Memristor Equations: Incomplete Physics and Undefined Passivity/Activity

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In his seminal paper, Chua presented a fundamental physical claim by introducing the memristor, "The missing circuit element". The memristor equations were originally supposed to represent a passive circuit element because, with active circuitry, arbitrary elements can be realized without limitations. Therefore, if the memristor equations do not guarantee that the circuit element can be realized by a passive system, the fundamental physics claims about the memristor as "missing circuit element" loses all its weight. The question of passivity/activity belongs to physics thus we incorporate thermodynamics into the study of this problem. We show that the memristor equations are physically incomplete regarding the problem of passivity/activity. As a consequence, the claim that the present memristor functions describe a passive device lead to unphysical results, such as violating the Second Law of thermodynamics, in infinitely large number of cases. The seminal memristor equations cannot introduce a new physical circuit element without making the model more physical such as providing the Fluctuation–Dissipation Theory of memristors.

Keywords: Memristors; incomplete models; activity; passivity.

1. Introduction

1.1. *On Chua's memristor model*

In 1971, Chua introduced the memristor, a new mathematical circuit theory element [1] that, to be fundamentally interesting, was supposed to be a passive device.

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The mathematical model interrelates the time-integral of the voltage (voltage flux) on the memristor

$$\Phi = \int_0^t U(\tau)d\tau \tag{1}$$

with the time-integral of the current (charge) through the device

$$q = \int_0^t I(\tau)d\tau \tag{2}$$

as follows:

$$M(q) = \frac{d\Phi}{dq}, \tag{3}$$

$$U(t) = M[q(t)]I(t), \tag{4}$$

where $M(q)$ is the memristor function see Fig. 1. It is claimed that for the memristor to be a *passive* circuit element the following condition must be satisfied:

$$M(q) \geq 0. \tag{5}$$

(Note, the lower boundary of the integral is often taken from minus infinity, which is unphysical. The more physical time coordinate "zero" can be the moment of time when the memristor comes into existence, and an initial value of the integral can be given, if needed. However, this issue has no importance in our present paper).

If the memristor function is constant, M_0, then the memristor and Eq. (4) represents a resistor of resistance R_0, that is, $M(q) = M_0$, where $R_0 = M_0$. Then, Eq. (4) becomes

$$U(t) = R_0 I(t). \tag{6}$$

One of the most fundamental questions about Chua's memristor model is:

Do Eqs. (1)–(5) *indeed always represent a passive device, or there are situations where the realization of a memristor function requires an active device?*

With active circuitry, arbitrary elements can be realized without limitations. Therefore, *if the memristor equations do not guarantee that the circuit element can be realized by a passive system, the fundamental physics claim about the memristor as "the missing circuit element" loses all its weight.*

It is important to note that many nonlinear impedances with memory effects exist since the existence of electronics that are passive devices and can be called memristors (for recent ones, see e.g., [2, 3]). These devices are not of our interest because they are examples for practical passive devices and they do not answer our fundamental physical question about the generality of the memristor claim.

The question of passivity/activity is a physical problem and former engineering definitions of this matter were shown self-contradictory and unphysical at certain practical conditions where they lead to perpetual motion machines [4]. Therefore, we will use the most advanced, statistical-thermodynamic definition of passivity/activity based on statistical physics that works correctly even with thermal noise (the quoted text below is from [4]):

(a) The device-in-question is active if the following condition holds. Suppose, we have a hypothetical-device, which does not require an external energy source to function and has the same signal-response characteristics as the device-in-question. In an isolated system with thermal equilibrium, such a hypothetical-device in a proper circuit would be able to produce steady-state entropy reduction in the system that is originally in thermal equilibrium, where the other elements are all passive. In other words, such hypothetical device would violate the Second Law of Thermodynamics.
Note for example, such an entropy reduction could be a steady-state positive or negative temperature gradient in the device's environment; etc.; anything that can break thermal equilibrium conditions and persists over the duration of operation of the device.
(b) Physical implication: Such a hypothetical-device cannot exist in practice, thus an active physical device always requires an external energy source to execute its response characteristics of activity.
(c) The device-in-question is passive if it is not active.

The structure of the paper is as follows:

In Sec. 2, we briefly outline the Fluctuation–Dissipation Theorem (FDT) of classical statistical physics.

In Sec. 3, we show why the above memristor equations are only abstract circuit theoretical models that are unphysical and/or incomplete in their present form and that the present model requires an active device in an infinite number of situations including the case when the memristor emulates a resistor.

2. FDT and Thermal Noise of Impedances and Resistors

Contrary to Chua's memristor model (Eqs. (1)–(6)), the statistical thermodynamics of the three fundamental circuit elements, that is, the resistor, capacitor and inductor, is sufficiently defined via the FDT [5–8]. It states that, in the classical

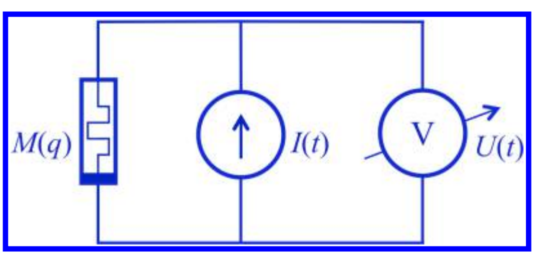

Fig. 1. Memristor driven by a current generator while the voltage on it is monitored. For definitions, see Eqs. (1)–(5) in the text.

physical limit, any impedance combined from these elements will act as a thermal noise generator with an internal impedance value identical to this particular impedance.

The Second Law of Thermodynamics requires [5–8] that, in thermal equilibrium, the time-average of the instantaneous power flow between two parallel impedances is zero, i.e.,

$$\langle P_{a \to b}(t,T) \rangle_t = 0, \tag{7}$$

where t is time and $P_{a \Longleftrightarrow b}(t,T)$ denotes instantaneous power flow between resistors Z_a and Z_b, as illustrated in Fig. 2. This expression holds for any frequency range.

For a *passive* impedance $Z(f)$ in thermal equilibrium, Eq. (7) demands that the functional form of the power density spectral function of noise voltage $U(t)$ is [8]:

$$S_u(f,T) = \mathrm{Re}[Z(f)]Q(f,T) = R(f)Q(f,T), \tag{8}$$

where $R(f)$ is the real part of the impedance and $S_u(f)$ represents the power density spectrum of thermal (Johnson) noise voltage. $Q(f,T)$ is a universal function of frequency and temperature and it is independent of material properties, geometry, and mechanism for electrical conduction [5–8].

In the low-frequency (classical physical limit):

$$Q(f,T) = 4kT. \tag{9}$$

For active impedances, there is no related fundamental law and the noise is determined by also other factors not visible in Eq. (8).

For inductors and capacitors, Eq. (8) implies zero thermal noise. For a resistor, Eqs. (8) and (9) imply that the voltage noise spectrum is $S_U(f,T) = 4kTR$ and its Norton-equivalent, the thermal noise spectrum is $S_I(f,T) = 4kT/R$.

Let us assume that the impedance is a resistor R_0. Then, the current driven resistor will produce the following voltage:

$$U(t) = R_0 I(t) + U_n(t), \tag{10}$$

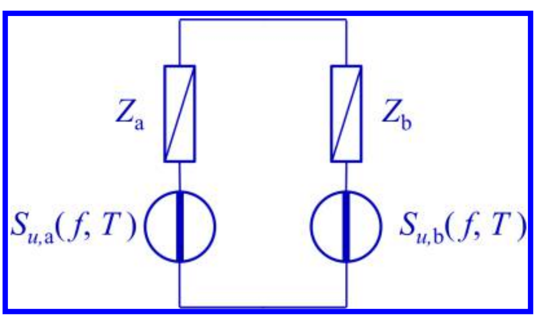

Fig. 2. Circuit for discussing Johnson noise and representing net power flow between resistors Z_a and Z_b. The voltage generators produce $S_u(f,T)$ power density spectra at frequency f and temperature T.

where $U_n(t)$ is a Gaussian thermal noise with mean-square value $\int_{f_L}^{f_H} 4kTR_0 df$, where f_L and f_H are the low and high frequency cutoffs of the frequency band of observation, respectively.

3. The Memristor Equations are Incomplete and Unable to Define a Physical Device

3.1. *The case of linear memristors*

There is a striking difference between Eqs. (6) and (10). Equation (6) with zero thermal noise for the memristor emulating a resistor is unphysical unless the memristor is an active device or, if it is passive, it must be at zero Kelvin absolute temperature to have zero noise. (Note, even the assumptions that an active impedance can have zero noise or that a physical body has zero absolute temperature are unphysical but let us disregard this problem).

To prove the above statement, let us suppose that the memristor is passive. Then, in the arrangement shown in Fig. 2, suppose that Z_a and Z_b are a memristor and a resistor, defined by Eqs. (6) and (10), respectively and both are at the same temperature T. As it is well-known, it follows from Eqs. (6) and (8)–(10), that in the frequency band $[f_L, f_H]$, the mean power flow between two identical resistors of R_0 at temperatures T_1 and T_2 is given as (e.g., [5–7]):

$$\langle P_{1\to 2}\rangle_t = 4k(T_1 - T_2)(f_H - f_L)\frac{R_0^2}{4R_0^2} = k(T_1 - T_2)(f_H - f_L). \tag{11}$$

Then, if the memristor model provides a full description, there would be a nonzero mean power flow $\langle P_{R\to M}(T)\rangle_t$ from a passive resistor at temperature $T > 0$ to a parallel memristor with zero thermal noise at the same temperature:

$$\langle P_{R\to M}(T)\rangle_t = kT(f_H - f_L) > 0. \tag{12}$$

However, both the memristor and the resistor are at the same temperature T therefore a nonzero power flow violate the Second Law of Thermodynamics unless there is an external energy source that supplies this power. *The physical conclusion is* [4], *that a memristor described by Eq.* (6) *must be an active device.*

Alternatively, if the given memristor is actually a passive device then we must realize that the memristor equations are incomplete to address the statistical thermodynamics of the system. The statistical physics (FDT) of the memristor is undefined and, without that, Eqs. (1)–(6) are not only incomplete but they are also incorrect if they are used to decide about the passivity/activity of the memristor.

3.2. *The case of nonlinear memristors*

The violation of the Second Law due to assuming passivity of the memristors and the validity of Eqs. (1)–(5) goes much beyond the case of linear memristors. Both from the above considerations and from [4], it is obvious, that assuming a thermal

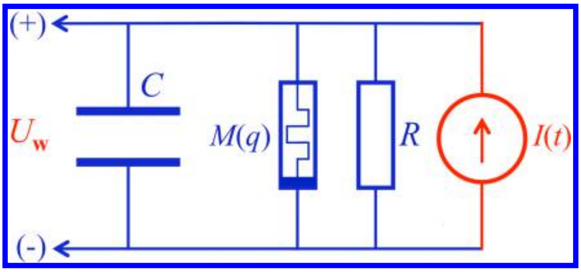

Fig. 3. A DC voltage source by a rectifying-type memristor with the circuit model of the thermal noise current $I(t)$ of a shunt resistor.

noise-free memristor, as Eqs. (1)–(6) do, will break the Second Law whenever the memristor function describes a dissipative device. Then, similarly to the linear case above, connecting a resistor parallel to the memristor will transfer non-zero net thermal noise power to the memristor from the resistor. As a consequence, the resistor is cooled and the memristor is heated.

Here, as the demonstration of infinite number of cases of activity, instead of investigating the general question of dissipativity of memristor functions, we point out that a smaller yet still infinitely large number of cases exists when the nonlinear memristor described by Eqs. (1)–(5) must be active: the case when an asymmetric memristor function shows rectifying characteristics, even at the slightest level, see Fig. 3.

If the thermal noise current flowing through the memristor and due to the proper asymmetry of the Φ and M functions, the U_w voltage on the capacitor has non-zero

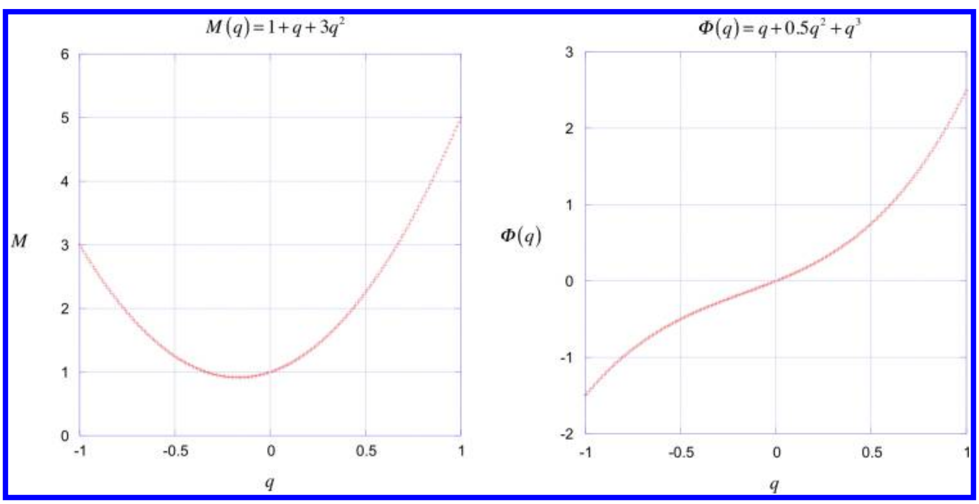

Fig. 4. Memristor functions with rectifying characteristics. The asymmetry of M provides an asymmetry in Φ and a nonzero DC component in U_w when driven by an AC current. In physical cases, such M and Φ functions will be bounded at higher values however it is unnecessary to deal with this issue for thermal noise, which is the weakest signal in the system.

mean (DC) value, even if it has only an infinitesimally small DC component, a perpetual motion machine can be built. The same situation for an unphysical (that is, noise-free) diode was already pointed out by Brillouin [9].

We gave a proof of Brillouin's paradox in [4] by showing that putting a large number N of circuits shown in Fig. 3 into a series circuitry results in an unbounded available power proportional to N due to the DC components that are always positively correlated. In conclusion, any passive circuitry that can rectify thermal noise, even in the slightest way, in unphysical because it breaks the Second Law. Such circuit must contain an active element.

As an illustrative example, we created simple memristor functions that are able to rectify, see Fig. 4. It is obvious that these simple examples represent an infinite set of rectifying memristor functions

$$\Phi(q) = aq + bq^2 + cq^3, \tag{13}$$

if we choose the coefficients of q so that Eq. (5) is satisfied and each of these coefficients are greater than zero.

4. Conclusion

In their present noise-free form, the memristor Eqs. (1)–(5) require the presence of an active device in infinitely many cases of memristor functions.

Chua's general proof that the memristor is passive based on Eqs. (1)–(5) is invalid because the memristor equations do not provide a complete description of the physics of memristors. A relevant statistical thermodynamics is needed for the memristors however the memristor equations do not contain sufficient physics to deduce that.

While the response functions of passive resistors, capacitors and inductors determine their FDT, the memristor model lacks sufficient physics for a FDT. To create the noise theory (FDT) of a memristor, it is essential to know the internal material structure of the particular memristor thus its general formulation, like it is done with the three basic circuit elements, is impossible.

Finally, we note that, in a similar, nonlinear situation Nico van Kampen had already showed [10] that the vacuum diode equations lead to unphysical results if the thermal noise of the diode is not included. He deduced the correct theory of noise in freestanding diodes, which is a relevant illustration of the essentially missing component of memristor theories.

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